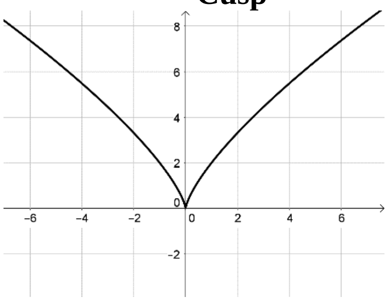
Math 1314
Lesson 13
Analyzing Other Types of Functions

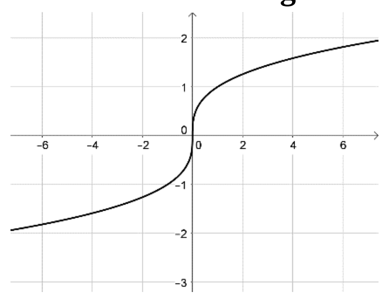
When working with functions different from polynomials, the critical numbers are defined as follows: The **critical numbers** of a function are numbers in the domain of the function where $f'(x) = 0$ or where $f'(x)$ is undefined.

The derivative is undefined whenever a function has a cusp, vertical tangent, hole, vertical asymptote, or jump discontinuity. Examples are shown below.

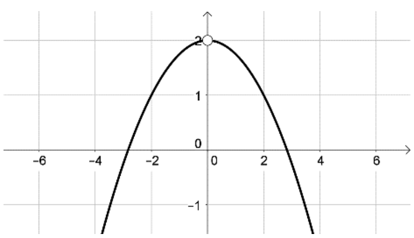
Cusp



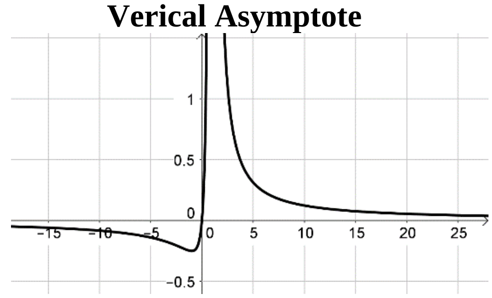
Vertical Tangent



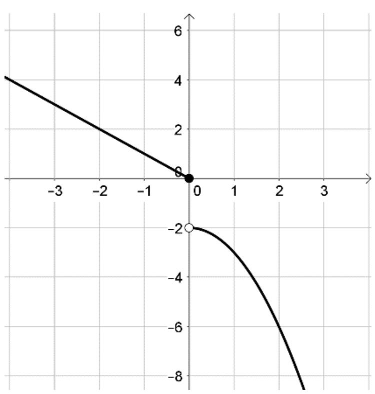
Hole



Verical Asymptote

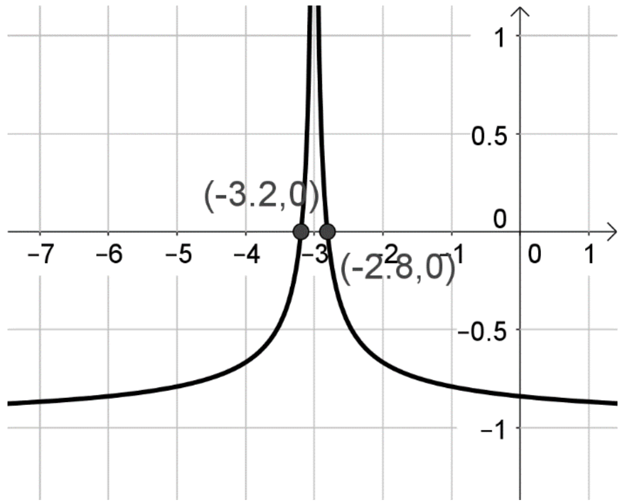


Jump Discontinuity

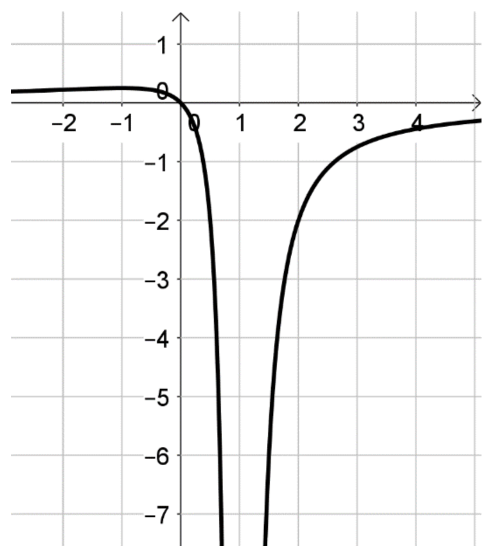


You must use caution when the graph of the derivative of a function is undefined since if this point is in the domain of the function*, it is a critical number.*

Example 1: The graph shown below is the graph of the derivative of $f(x)=\sqrt[3]{x+3}-x$. Give the function's domain and any critical numbers.



Example 2: The graph shown below is the graph of the derivative of a function *f*. The original function's domain is $(-\infty,1)\cup(1,\infty)$. Find any critical numbers.



Next we'll analyze other types of functions almost the same way we analyzed polynomial functions in Lesson 12.

Recall the commands are slightly different:

- Roots[<Function>, <Start x-Value>, <End x-Value>]
- Extremum[<Function>, <Start x-Value>, <End x-Value>]
- Asymptote[<function>]
- Inflectionpoint[<polynomial>] will NOT WORK FOR FUNCTIONS DIFFERENT FROM POLYNOMIALS. So we'll simply analyze the second derivative of a function to find any points of inflection.

In Lesson 12 we learned:

- How to find critical numbers. In this lesson, to find critical numbers we'll need to find when $f'(x) = 0$ or when $f'(x)$ is undefined. Then:

If	**Then**
$f' > 0$	f is increasing
$f' < 0$	f is decreasing

From here we can find any relative extrema.

A relative maximum or a relative minimum can only occur at a **critical number**.

- A function has a relative maximum if it changes from increasing to decreasing across a critical number.
- A function has a relative minimum if it changes from decreasing to increasing across a critical number.

If	**Then**
$f'' > 0$	f is concave up
$f'' < 0$	f is concave down

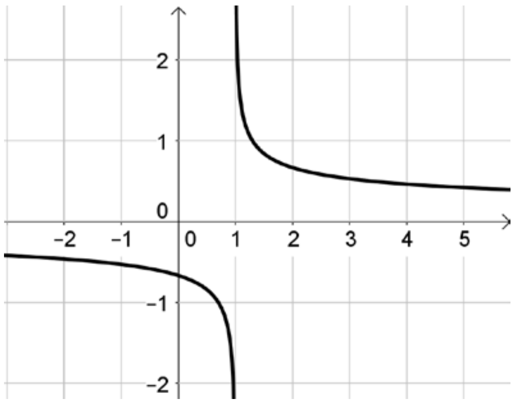
A point where concavity changes is a ***point of inflection***. This point must be in the domain of the function. In order to find the candidates of points of inflection, we'll need to find when:

- $f''(x) = 0$ or $f''(x)$ is undefined.

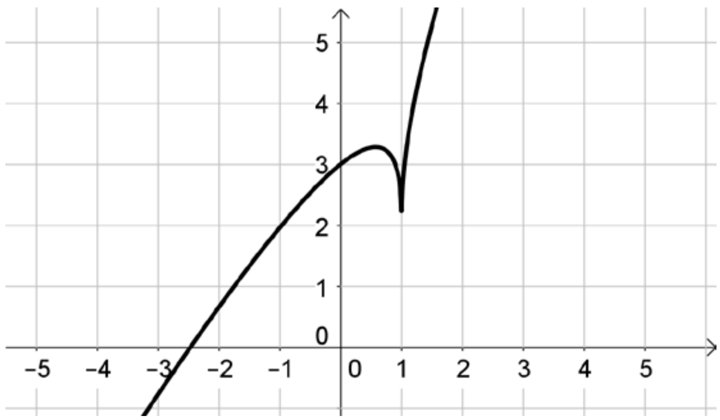
In fact if we were given the graph of the first derivative of a function, we could find all of the features above since:

Wherever	**Then**
f' is increasing	f is concave up
f' is decreasin	f is concave down

Example 3: The graph below is the graph of the first derivative of a function whose domain is $(-\infty, \infty)$. Find any critical numbers, relative extrema and any points of inflection.



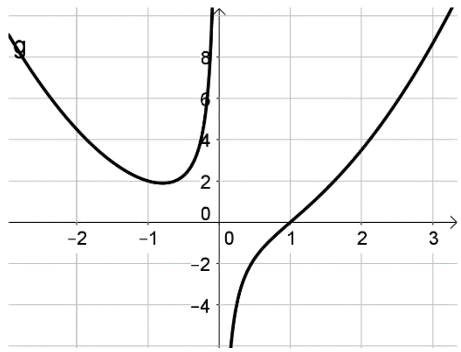
Example 4: Let $f(x) = 3(x-1)^{2/5} + 2x$ Find the function's domain, then find where the function is increasing/decreasing and any relative extrema. *Enter the function into GGB.*



Commands:

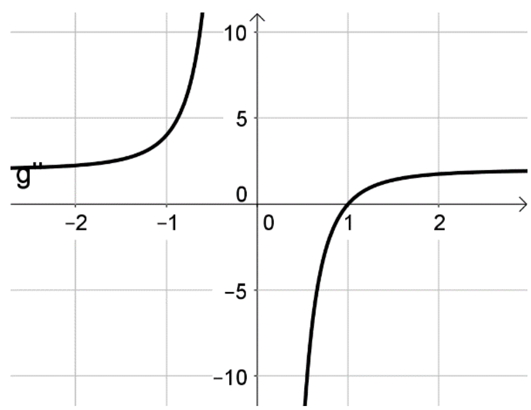
Example 5: Let $g(x) = x^2 - \frac{1}{x}$ find the function's domain, any roots, any asymptotes, intervals of increase/decrease, relative extrema, any intervals of concavity, and any points of inflection. *Enter the function into GGB.*

Commands:



We'll analyze the second derivaitve of the function to find any intervals of concavity and any points of inflection.

Commands:



Example 6: Let $f(x) = (x^2 - 2x)e^x + 1$. Find the function's domain, any intervals of concavity, and any points of inflection.

Enter the function into GGB. The graph below is the graph of the second derivative of the function f.

Commands: