Math 1314
Lesson 14
Optimization

When you optimize something, you make it as large as possible or as small as possible under certain stated conditions. Business owners wish to make revenues and profits as large as possible, while keeping costs as small as possible.

Absolute Extrema
If $f(x) \leq f(c)$ for all x in the domain of f, then f(c) is called the **absolute maximum value** of f.
If $f(x) \geq f(c)$ for all x in the domain of f, then f(c) is called the **absolute minimum value** of f.

So the y value of the point that has the biggest y value in the interval of interest is the absolute maximum value. The y value of the point that has the smallest y value in the interval of interest is the absolute minimum value. We can find absolute extrema by looking at a graph of a function.

Example 1: Find the absolute maximum and absolute minimum values of $f(x) = 3 + 4x^2 - x^4$.
Enter the function in GGB.
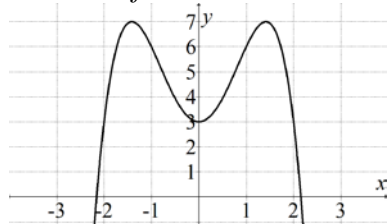
Absolute Maximum Value   Absolute Minimum Value

Example 2: Find the absolute maximum and absolute minimum values of the function
$f(x) = (x+1)^{2/3} - \frac{1}{3}x$ over the interval $(-\infty, 28.9008]$ *Enter the function in GGB.*
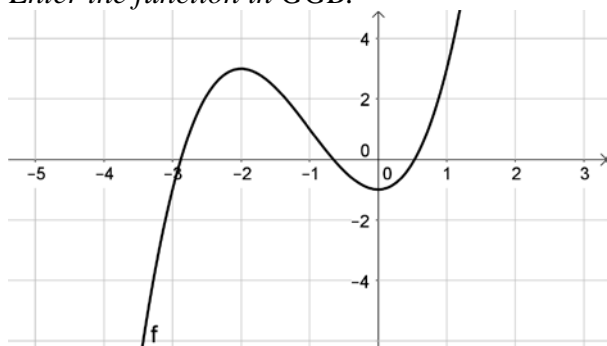

Absolute Maximum Value   Absolute Minimum Value

Finding the Absolute Extrema of *f* on a Closed Interval

1. Produce the graph of the function in GGB. Find the critical points of *f* that lie in (*a*, *b*) either by inspection or the appropriate command. *Recall: Critical points are x-values in the domain of the function, where the derivative is equal to 0 or undefined.*

2. Compute the value of the function at every critical point found in step 1 and also at the endpoints of the given closed interval.

3. The absolute maximum value will be the largest value found in step 2, and the absolute minimum value will be the smallest value found in step 2.

Example 3: Find the absolute extrema of $f(x) = x^3 + 3x^2 - 1$ over [-3, 1].
Enter the function in GGB.
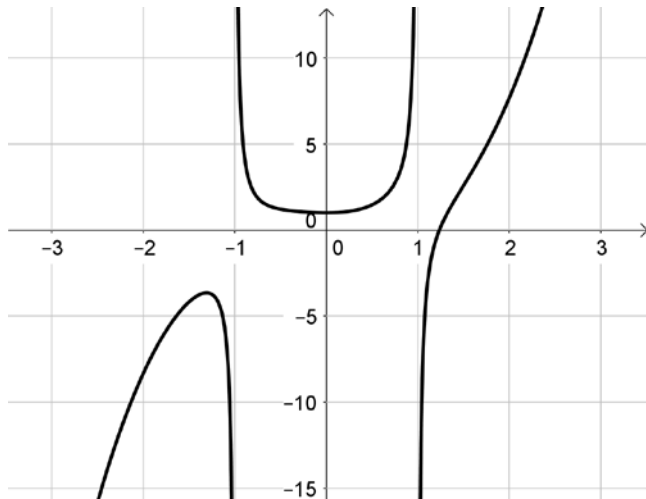

Commands:

Absolute Minimum Value: Absolute Maximum Value:

Example 4: Find the absolute extrema of the function $f(x) = x^3 - \dfrac{1}{x^2 - 1}$ on [-0.25, 1.25]. *Enter the function in GGB.*

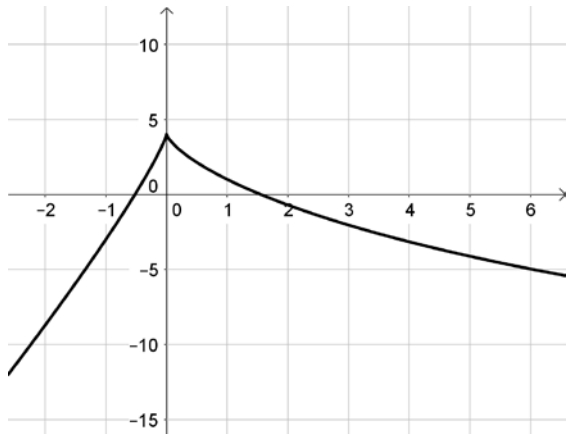Commands:

Absolute Minimum Value: Absolute Maximum Value:

Example 5: Find the absolute extremum of the function $f(x) = 2x - 5x^{4/5} + 4$ on [-1, 4].
Enter the function in GGB.

Commands:

Absolute Minimum Value: Absolute Maximum Value:

Applications of Absolute Extrema

Example 6: The cost of manufacturing x fishing rods is given by $C(x) = 500 + 7x + \dfrac{1}{10000}x^2$.

The price of each rod is a function of the number manufactured and is given by $p = 10 - 0.0004x$. If all fishing rods manufactured can be sold, what is the number of rods that yields a maximum profit? *Produce the profit function and enter it into GGB.*
Recall: $P(x) = R(x) - C(x)$, *where* $R(x) = px$

Commands: