

**Lesson 22**  
**Functions of Several Variables**

So far, we have looked at functions of a single variable. In this section, we will consider functions of more than one variable.

**Functions of Two Variables**

Definition: A real valued function of two variables,  $f$ , consists of a set  $A$  of ordered pairs of real numbers  $(x, y)$  called the domain of the function, and a rule that associates with each ordered pair in the domain of  $f$  one and only one real number, denoted by  $z = f(x, y)$ .

You will need to learn two skills using functions of several variables: Evaluating at a given point and determining the domain.

Example 1: Suppose  $f(x, y) = xe^{2x} - 5xy^2 + \ln(xy)$  Compute  $f(-1, -3)$ .  
*Enter the function as shown in GGB.*

Command:

Answer:

Example 2: The volume of a cylindrical tank with radius  $r$  and height  $h$  is given by the formula  $f(r, h) = \pi r^2 h$ . Find the volume of a tank with radius 6 feet and height 20 feet.  
*Enter the function as shown in GGB.*

Command:

Answer:

Example 3: The monthly payment that amortizes a loan of  $A$  dollars in  $t$  years when the interest rate is  $r$  per year is given by

$$P = f(A, r, t) = \frac{Ar}{\left(12 \left(1 - \left(1 + \frac{1}{12}r\right)^{-12t}\right)\right)}$$

Find the monthly payment for a mortgage of \$250,000 that will be amortized over 25 years with an interest rate of 4.5% per year.  
*Enter the function as shown in GGB.*

Command:

Answer:

Example 4: Find the domain of the function:  $f(x, y) = 2x^2 + 3y^2$

Example 5: Find the domain of the function:  $f(x, y) = \frac{3x}{2x - 5y}$

Example 6: Find the domain of the function:  $f(x, y) = \sqrt{16x - y}$