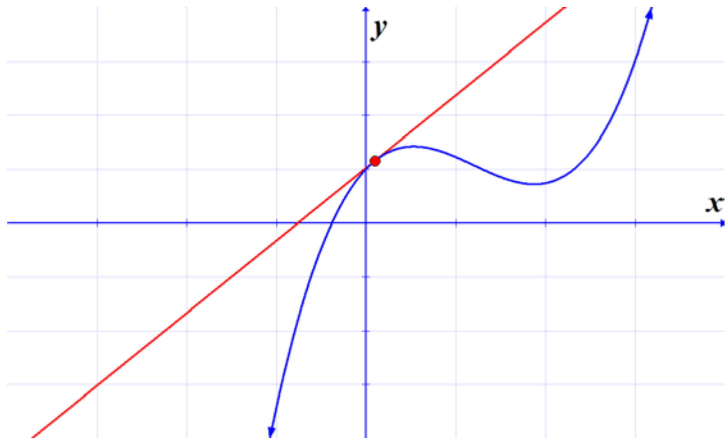
Math 1314
Lesson 7
Applications of the Derivative

Recall from Lesson 6 that the derivative gives a formula for finding the slope of the tangent line to a function at any point on that function.



Example 1: Find the slope of the tangent line of $f(x) = x^{\frac{3}{2}} - e^{2x} + \ln(x)$ when x = 4.
Enter the function into GGB.
Command: Answer:

Equation of the Tangent Line

We can also find the equation of the tangent line by using the following GGB command.
Command: tangent[<x-Value>,<Function>]

Example 2: Give the equation of the line tangent to $f(x) = 1.6x^3 + 6.39x - 2.81$ at (3, 59.56). *Enter the function into GGB.*
Command: Answer:

In other cases, we may want to find all values of *x* for which the tangent line to the graph of *f* is horizontal. Since the slope of any horizontal line is 0, we'll want to find the roots of the derivative.

Example 3: Find all *x*-values on the graph of $f\left(x\right)=5-3x+2x^2$ where the tangent line is horizontal.

We can also determine values of *x* for which the derivative is equal to a specified number. Set the derivative equal to the given number and solve for *x*.

Example 4: Find all values of *x* for which $f\,'(x)=3$: $f(x)=\frac{1}{3}x^3+\frac{5}{2}x^2-7x+3$

Example 5: Find the value of the second derivative when *x* = 5 if $f(x)=\frac{x^2\ln x}{(x^2+3)^{\frac{1}{3}}}$.

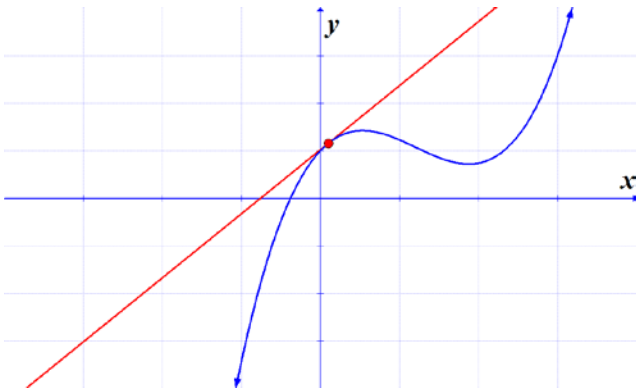
Enter the function into GGB.
Command: Answer:

Next we'll work through some word problems. In many of our word problems, we'll be asked for either:

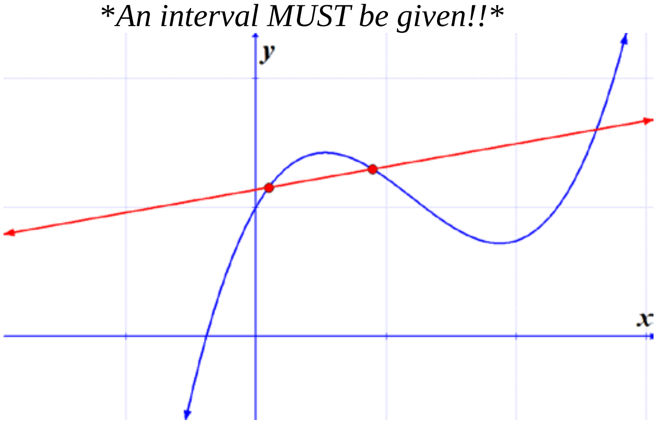
- a function value.
- the rate at which something is changing at a specific number.
- average rate of change.

In word problems, whenever it's anything about a(n):

- *slope of a tangent line, rate of change, instantaneous rate of change, velocity----this will imply that we need the derivative*



- *average rate of change, difference quotient this will imply we need an average.*
  An interval MUST be given!!



Position, Velocity and Acceleration

A **position function**, $f(t)$, gives the position of an object with respect to time (that's why we use t instead of x).

Assuming its derivative exists, the derivative of the position function is the **velocity function** $f'(t)$. This will give the rate of change at time t (how fast the position is changing or the instantaneous rate of change). We refer to the absolute value of velocity as **speed**. Velocity has two components: speed and direction.

Velocity can be positive, negative or zero.

Vertical motion:

- If you throw a rock up in the air, its velocity will be positive while it is moving upward and will be negative while it is moving downward.

Horizontal motion:

- If the velocity is positive, the object is moving right (positive direction). *Here the position is increasing.*
- If the velocity is negative, the object is moving left (negative direction). *Here the position is decreasing.*
- If the velocity is zero, the object has stopped.

Assuming the velocity function is differentiable, **acceleration** is defined to be the rate at which an object changes its velocity per unit time (how fast the velocity changing). Usually denoted by $f''(t)$.

Acceleration can only occur when changing speed, direction or both.

- Positive acceleration corresponds to accelerating (increasing velocity).
- Negative acceleration corresponds to decelerating (decreasing velocity).

Example 6: Suppose the distance covered by a car can be measured by the function $f(t) = 4t^2 + 32t$, where $f(t)$ is given in feet and t is measured in seconds.

a. Find the distance covered by the car in 10 seconds.

b. Find the rate of change of the car when $t = 4$. Is the car moving left or right?

c. Find the average velocity of the car over the interval [1, 6].

Recall: $\dfrac{f(t+h) - f(t)}{h}$.

Free Fall

In free fall (upward motion or downward motion), the height of an object is given by $h(t) = -16t^2 + v_0 t + h_0$ (distance in feet) or $h(t) = -4.9t^2 + v_0 t + h_0$ (distance in meters), where v_0 is the initial velocity, h_0 is the initial height and *t* is time.

Example 7: A person standing on top of a building that is 112 feet high. This person throws a rock vertically upward with an initial velocity of 84 feet per second.

a. When does the rock hit the ground and what is its velocity on impact?

Commands: Answers:

b. Is the object rising or falling when $t = 3$?

Command: Answer:

c. When is the velocity zero?

Command: Answer:

d. What is the acceleration at any time *t*?

Commands: Answers:

Example 8: An object is dropped from a certain height in meters and hits the ground 5 seconds later. From what height was the object dropped?

Example 9: A study conducted for a specific company showed that the number of lawn chairs assembled by the typical worker t hours after starting work at 6 a.m. is given by $N(t) = -t^3 + 7t^2 + 18t$.

a. At what rate will the typical worker be assembling lawn chairs at 9 a.m.?
Command: Answer:

b. How many lawn chairs will the typical worker have assembled by 12 p.m.?
Command: Answer:

c. What is the average rate at which the lawn chairs are assembled from 7 a.m. to 11 a.m.

Command: Answer:

Example 10: The median price of a home in one part of the US can be modeled by the function $P(t) = -0.01363t^2 + 9.2637t + 125.84$, where $P(t)$ is given in thousands of dollars and t is the number of years since the beginning of 1995. According to the model, at what rate were median home prices changing at the beginning of 2005?

Command: Answer:

Example 11: A country's gross domestic product (in millions of dollars) is modeled by the function $G(t) = -2t^3 + 45t^2 + 20t + 6000$ where $0 \le t \le 11$ and $t = 0$ corresponds to the beginning of 1997. What was the average rate of growth of the GDP over the period 1999 – 2004?

Command: Answer: