Section 8.1
Parabolas

We previously studied parabolas as the graphs of quadratic functions. Now we will look at them as conic sections. There are a few differences. For example, when we studied quadratic functions, we saw that the graphs of the functions could open up or down. As we look at conic sections, we'll see that the graphs of these second degree equations can also open left or right. So, not every parabola we'll look at in this section will be a function.

A **parabola** is the set of all points equally distant from a fixed line and a fixed point not on the line. The fixed line is called the **directrix**. The fixed point is called the **focus**.

The axis, or **axis of symmetry**, runs through the focus and is perpendicular to the directrix. The **vertex** is the point halfway between the focus and the directrix.

Basic "Vertical" Parabola:

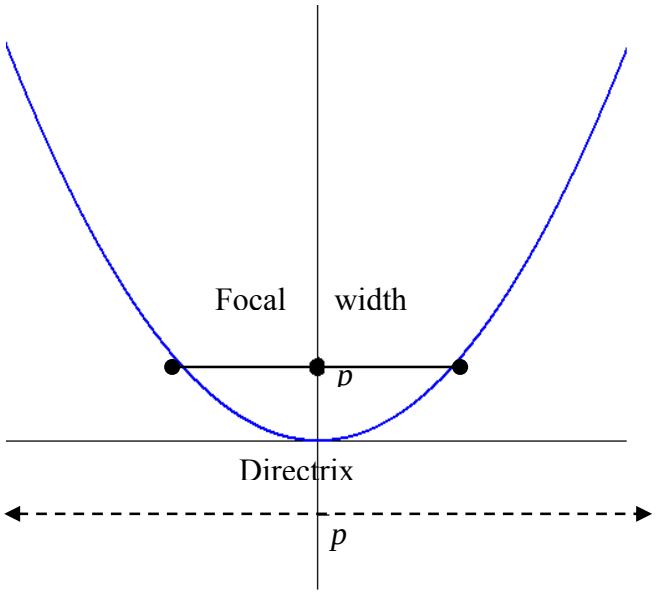
Equation: $x^2 = 4py$

Focus: $(0, p)$

Directrix: $y = -p$

Focal Width: $|4p|$

Coordinates of Focal Chord:
$(\pm 2p, p)$



Basic "Horizontal" Parabola:

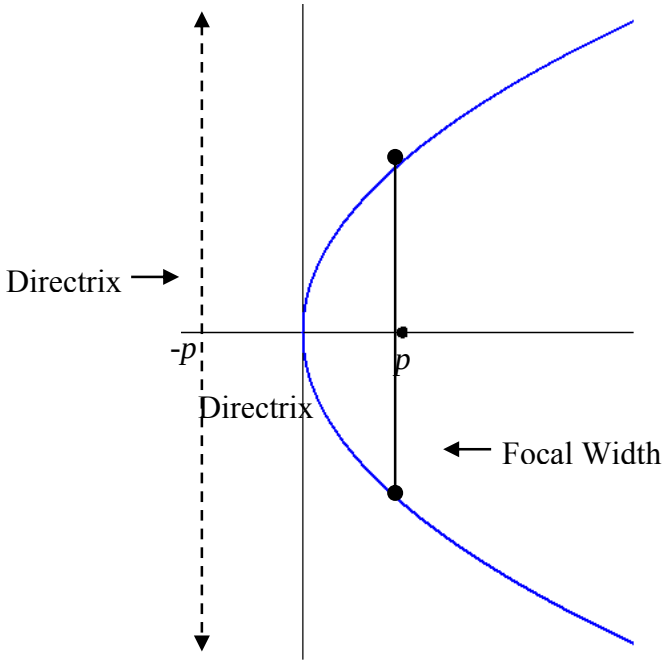
Equation: $y^2 = 4px$

Focus: $(p, 0)$

Directrix: $x = -p$

Focal Width: $|4p|$

Coordinates of Focal Chord:
$(p, \pm 2p)$



Graphing parabolas with vertex at the origin:

- When you have an equation, look for x^2 or y^2
- If it has x^2, it's a "vertical" parabola. If it has y^2, it's a "horizontal" parabola.
- Rearrange to look like $y^2 = 4px$ or $x^2 = 4py$. In other words, isolate the squared variable.
- Determine *p*.
- Determine the direction it opens.
  - If *p* is positive, it opens right or up.
  - If *p* is negative, it opens left or down.
- Starting at the origin, place the focus *p* units to the inside of the parabola. Place the directrix *p* units to the outside of the parabola.
- Use the focal width $4p$ ($2p$ on each side) to make the parabola the correct width at the focus.

Graphing parabolas with vertex not at the origin:

- Rearrange (complete the square) to look like $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$.
- Vertex is (h,k). Draw it the same way, except start at this vertex.

Example 1: Graph $6x^2 + 24y = 0$. Find vertex, focus, directrix, focal width, axis of symmetry and endpoints of the focal chord.

Example 2: Graph $y^2 + 2y + 8x + 17 = 0$. Find vertex, focus, directrix, focal width, axis of symmetry and endpoints of the focal chord.

Example 3: Suppose you know that the focus of a parabola is (-1, 3) and the directrix is the line $y = -1$. Write an equation for the parabola in standard form.

Example 4: Given: $x^2 - 2x - 4y + 9 = 0$. State the following, value for p, the equation of the directrix, the focal point, endpoints of the focal chord and graph the parabola.

Example 5: Match each equation to its graph.

a. $(x+2)^2 = 2(y-1)$ b. $(x+2)^2 = -2(y-1)$ c. $(y+2)^2 = -2(x-1)$