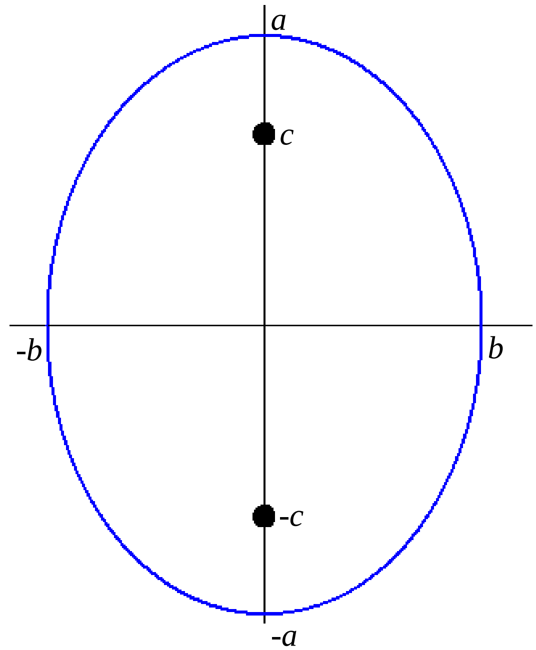
Math 1330 Section 8.2B

Section 8.2
Ellipses

An **ellipse** is the set of all points, the sum of whose distances from two fixed points is constant. Each fixed point is called a **focus** (plural = *foci*).

Basic "Vertical" Ellipse (centers at origin):



Basic "vertical" ellipse:

Equation: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a>b$

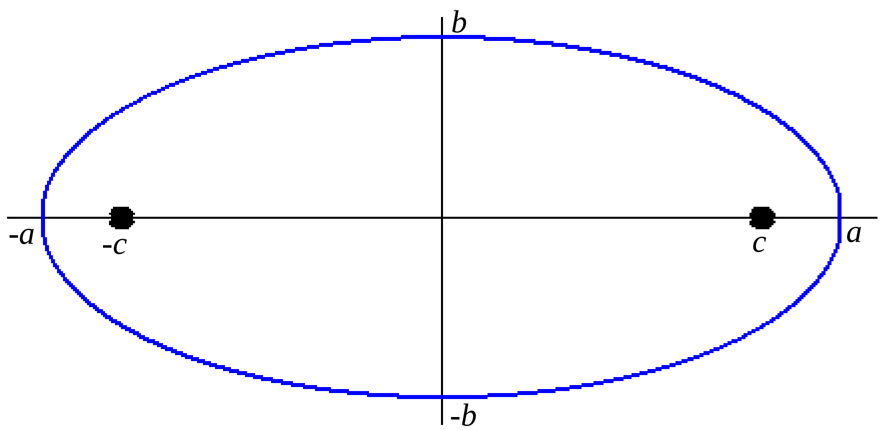
Foci: $(0,\pm c)$, where $c^2=a^2-b^2$

Vertices: $(0,\pm a)$

Eccentricity: $e=\frac{c}{a}$

The **eccentricity** provides a numerical measure of how much the ellipse deviates from being a circle. The *eccentricity e* is a number between 0 and 1.

Basic "Horizontal" Ellipse:



Equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$

Foci: $(\pm c,0)$, where $c^2=a^2-b^2$

Vertices: $(\pm a,0)$

Eccentricity: $e=\frac{c}{a}$

For ellipses, the line segment joining the vertices is called the **Major Axis (length 2a)** and the line segment through the center and perpendicular to the major axis with endpoints on the ellipse is called the **Minor Axis (length 2b)**.

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Graphing Ellipses

To graph an ellipse with center at the origin:

- Rearrange into the form $\frac{x^2}{number} + \frac{y^2}{number} = 1$.

- Decide if it's a "horizontal" or "vertical" ellipse.
  - if the bigger number is under x^2, it's horizontal (longer in *x*-direction).
  - if the bigger number is under y^2, it's vertical (longer in *y*-direction).

- Use the square root of the number under x^2 to determine how far to measure in *x*-direction.

- Use the square root of the number under y^2 to determine how far to measure in *y*-direction.

- Draw the ellipse with these measurements. Be sure it is smooth with no sharp corners

- $c^2 = a^2 - b^2$ where a^2 and b^2 are the denominators. (Subtract the small denominator from the large denominator to get c^2.)

- The foci are located *c* units from the center on the long axis.

To graph an ellipse with center not at the origin:

- Rearrange (complete the square if necessary) to look like $\frac{(x-h)^2}{number} + \frac{(y-k)^2}{number} = 1$.

Start at the center (h,k) and then graph it as before.

Example 1: Graph $\frac{x^2}{16} + \frac{y^2}{9} = 1$. Find the center, vertices, foci, lengths of Major and Minor Axes, the coordinates of the Major and Minor Axes and the eccentricity.

Example 2: Graph $4x^2 - 8x + 9y^2 - 54y = -49$. Find the center, vertices, foci, lengths of Major and Minor Axes, the coordinates of the Major and Minor Axes and the eccentricity.

Example 3: Find the equation for the ellipse satisfying the given conditions.
Foci are (-2, -3) and (-2, 5), a = 8.

Example 4: Find the equation for the ellipse satisfying the given conditions
Foci : $(0,4)$ and $(0,-4)$ and sum of foci radii is 10 .

Example 5: Given that foci $(-2,-1)$ and $(6.-1)$, passes through the point $(2,\ 2)$. Write the equation for the ellipse in standard form.

Example 6: Graph the ellipse and state the foci.

$$\frac{x^2}{25} + \frac{(y-1)^2}{36} = 1$$