Section 8.3
Hyperbolas

A **hyperbola** is the set of all points, the difference of whose distances from two fixed points is constant. Each fixed point is called a **focus** (plural = *foci*).

The **focal axis** is the line passing through the foci.

Basic "Vertical" Hyperbola:

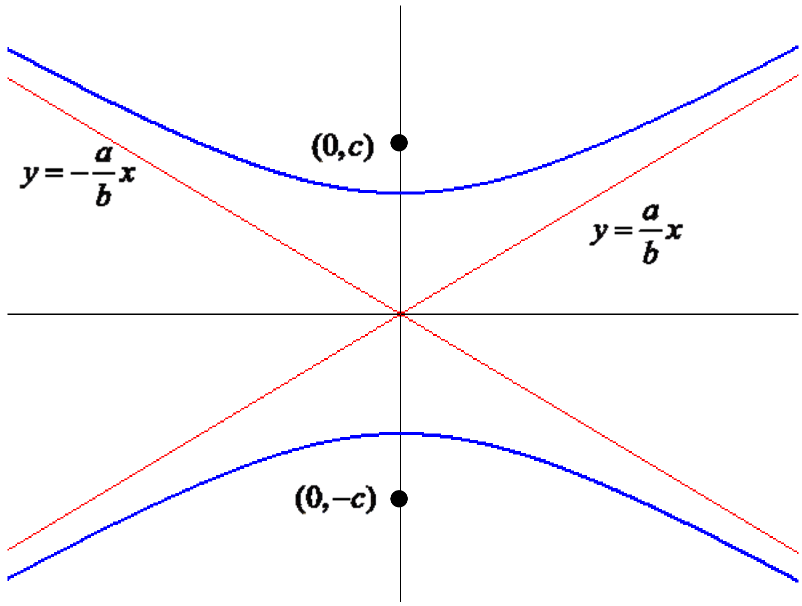
Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Asymptotes: $y = \pm\frac{a}{b}x$

Foci: $(0, \pm c)$, where $c^2 = a^2 + b^2$

Vertices: $(0, \pm a)$

Eccentricity: $e = \frac{c}{a}$



Basic "Horizontal" Hyperbola:

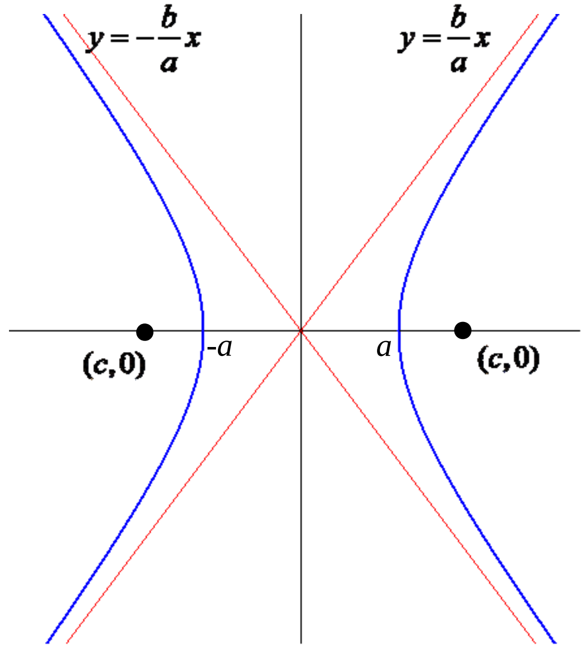
Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Asymptotes: $y = \pm\frac{b}{a}x$

Foci: $(\pm c, 0)$, where $c^2 = a^2 + b^2$

Vertices: $(\pm a, 0)$

Eccentricity: $e = \frac{c}{a}$



The **transverse axis (length 2a)** is the line segment joining the two vertices. The **conjugate axis (length 2b)** is the line segment perpendicular to the transverse axis, passing through the center and extending a distance *b* on either side of the center.

Math 1330 Section 8.3
Graphing Hyperbolas:

<u>To graph a hyperbola with center at the origin</u>:

- Rearrange into the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

- Decide if it's a "horizontal" or "vertical" hyperbola.
    - if x^2 is positive, it's horizontal (vertices are on *x*-axis).
    - If y^2 is positive, it's vertical (vertices are on *y*-axis).
- Use the square root of the number under x^2 to determine how far to measure in *x*-direction.

- Use the square root of the number under y^2 to determine how far to measure in *y*-direction.

- Draw a box with these measurements.

- Draw diagonals through the box. These are the asymptotes. Use the dimensions of the box to determine the slope and write the equations of the asymptotes.

- Put the vertices at the edge of the box on the correct axis. Then draw a hyperbola, making sure it approaches the asymptotes smoothly.

- $c^2=a^2+b^2$ where a^2 and b^2 are the denominators.

- The foci are located *c* units from the center, on the same axis as the vertices.

<u>To graph a hyperbola with center not at the origin</u>:

- Rearrange (complete the square if necessary) to look like

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \text{ or } \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1.$$

- Start at the center (h,k) and then graph it as before.
- To write down the equations of the asymptotes, start with the equations of the asymptotes for the similar hyperbola with center at the origin. Then replace *x* with $x-h$ and replace *y* with $y-k$.

Example 1: Graph $\frac{x^2}{36} - \frac{y^2}{4} = 1$. Find the center, vertices, foci, asymptotes, length and coordinates of both Transverse and Conjugate Axes and the eccentricity.

Example 2: Graph $y^2 - 25x^2 + 8y - 9 = 0$. Find the center, vertices, foci, asymptotes, length and coordinates of both Transverse and Conjugate Axes and the eccentricity.

Example 3: Given: $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$. Find the center, vertices, foci, asymptotes, length and coordinates of both Transverse and Conjugate Axes and the eccentricity

Example 4: Use the following information to write the equation for the hyperbola in standard form. Vertices: (2, 2) and (8, 2), b = 4

Example 5: Use the following information to write the equation for the hyperbola in standard form. Foci are $(-4, 0)$ and $(4, 0)$ and the length of conjugate axis is 6.