Systems: Identify Equations, Point of Intersection of Equations

Classification of Second Degree Equations

Recall the following equations:

Parabola: $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$.

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Ellipse: $\dfrac{(x-h)^2}{number} + \dfrac{(y-k)^2}{number} = 1$

Hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ or $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

Sometimes equations that look like they should be conic sections do not behave very well.

For example,

I. $(x-3)^2 + (y+1)^2 = 0$ represents a point (3, -1). Looks like it could be a circle equation, but r = 0.

II. $9x^2 - 4y^2 = 0$ represents 2 lines. Looks like it could be a hyperbola, but right hand-side is 0, not 1.

Solve for y:

$$4y^2 = 9x^2$$

$$y^2 = \frac{9x^2}{4}$$

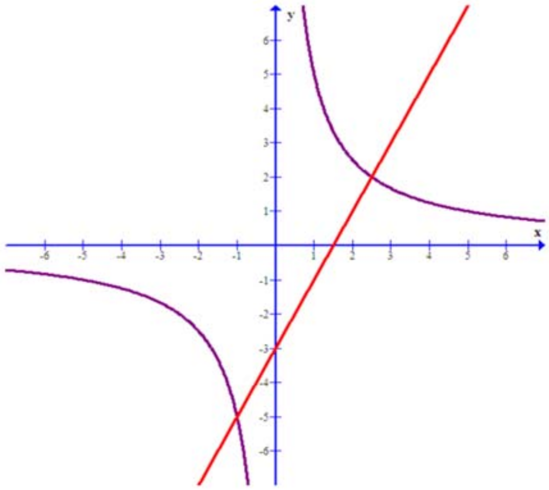
$$y = \pm\frac{3x}{2}$$

III. $2x^2 + 3y^2 = -1$ represents nothing, no graph, no point, no line(s). Looks like it could be an ellipse, but right hand-side is -1, not 1.

These are all examples of *degenerate conic sections*. You will not see these very often, but you should be aware of them.

Systems of Second Degree Equations

When we graph two conic sections or a conic section and a line on the same coordinate planes, their graphs may contain points of intersection. The graph below shows a hyperbola and a line and contains two points of intersection.



We want to be able to find the points of intersection. To do this, we will solve a system of equations, but now one or both of the equations will be second degree equations. Determining the points of intersection graphically is difficult, so we will do these algebraically.

Example 1: Identify each conic. (Note, some of these are degenerates conics.)

a. $12x = y^2$

b. $\dfrac{(x-2)^2}{9} - \dfrac{(y+2)^2}{16} = 1$

c. $\dfrac{(x+4)^2}{4} + \dfrac{(y-1)^2}{4} = 1$

d. $6x^2 - 4xy + 3y^2 + 5x - 7y + 3 = 0$

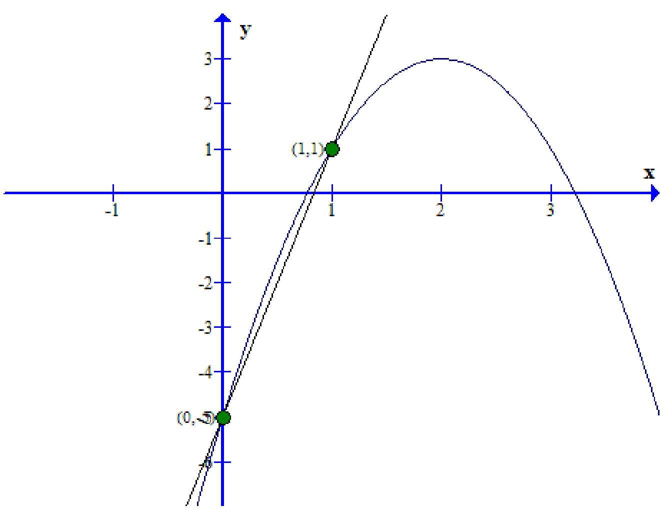
e. $2x^2 - 8y^2 = 0$

f. $(x+1)^2 + (y-1)^2 = 0$

g. $x^2 + 4y^2 = -8$

h. $y^2 + 12x + 2y - 23 = 0$

Example 2: Solve the system of equations:
$f(x) = -2x^2 + 8x - 5$
$g(x) = 6x - 5$

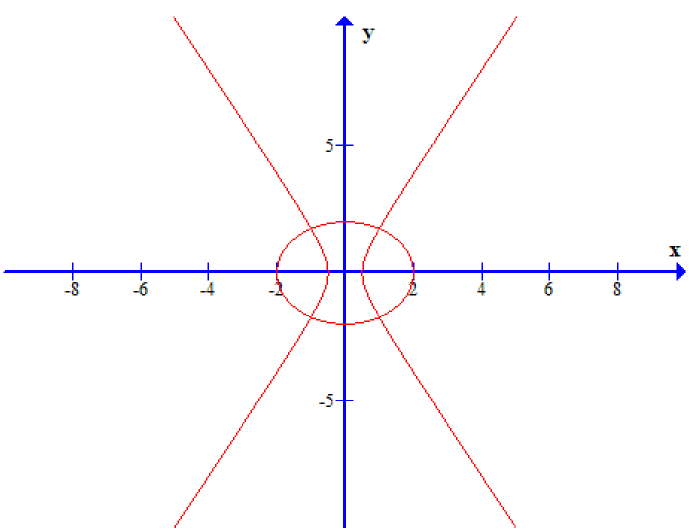


Solutions: $(0, -5)$ and $(1,1)$

Example 3: Solve the system of equations:

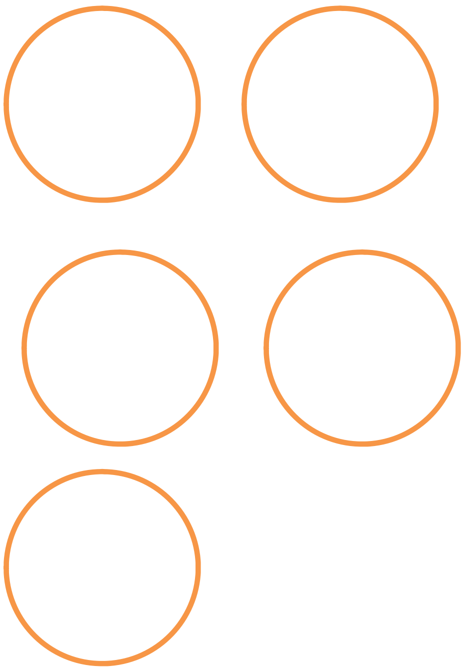
$$x^2 + y^2 = 4$$
$$4x^2 - y^2 = 1$$

Solutions: $(1, \sqrt{3})$, $(1, -\sqrt{3})$, $(-1, \sqrt{3})$, $(-1, \sqrt{3})$



Example: 4: Solve the system of equations: $x^2 + y^2 = 9$
$y = x^2 + 3$

Possible Solutions:

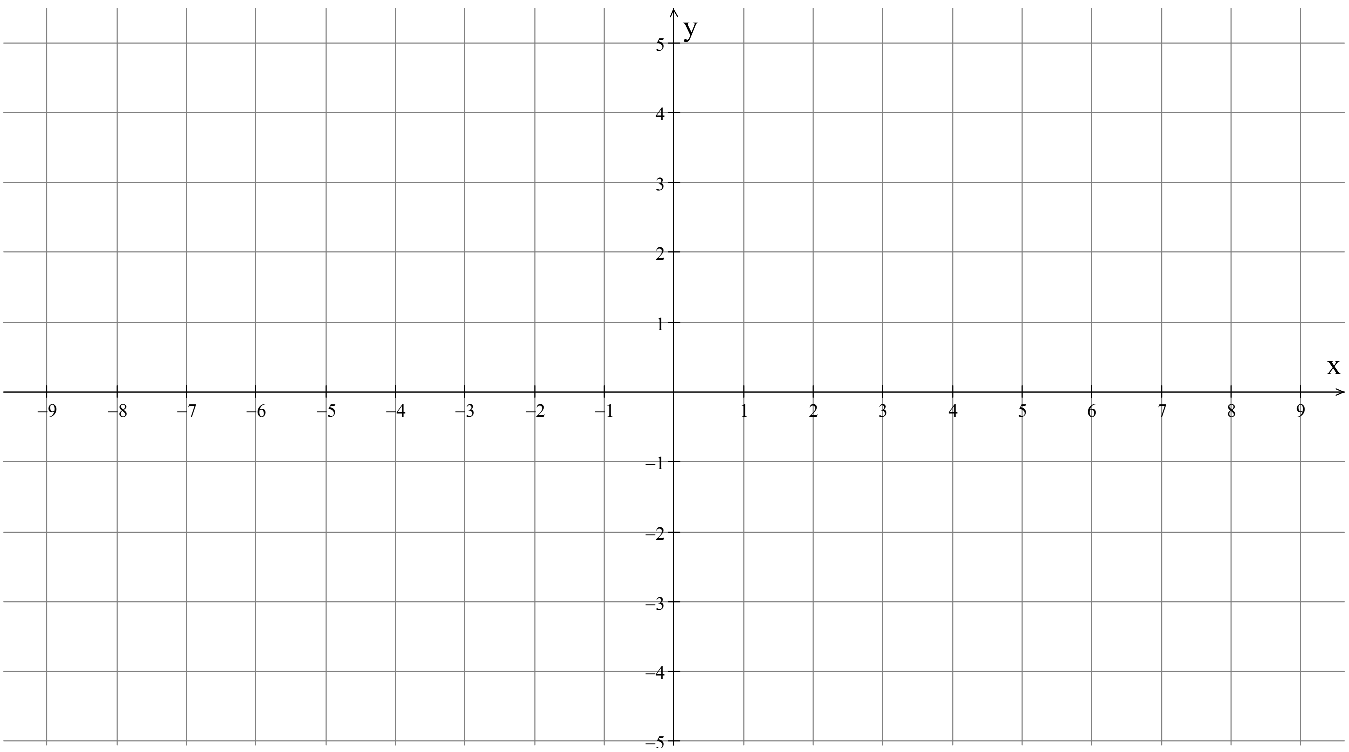


Example 5: Solve the system of equations:
$(x-1)^2 + (y-3)^2 = 4$
$y = x$

Example 6: Graph each equation and determine the number of points of intersection of the two graphs.

$$x^2 + y^2 = 36$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$



Example 7: Solve the system of equations:

$$x^2 + y^2 = 13$$
$$x^2 - y^2 = 7$$

Review of chapter 8:

Example 8: Find the coordinates of the center and radius for the given circle.

$$x^2 + y^2 + 4x - 6y - 23 = 0$$

Example 9: State the vertices eccentricity and equations of the asymptotes for the following;

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Example 10: Use the following function: $\frac{x^2}{4}+\frac{y^2}{16}=1$
State the vertices of the above function and state the length of the major axis.

Example 11: Given; $y^2=-4x$

a. Dose this parabola open upward, downward, to the left or to the right?

b. State the value for p.

c. State the equation of the directrix.

d. State the focal point.

e. State the endpoints of the focal chord.

f. Graph