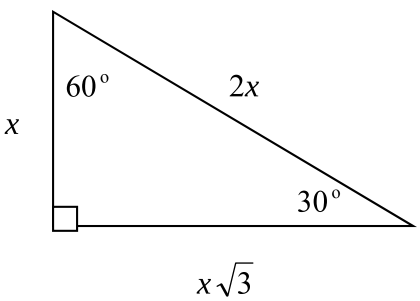
Section 4.1
Special Triangles and Trigonometric Ratios

In this section, we'll work with some special triangles before moving on to defining the six trigonometric functions.
Two special triangles **$30° - 60° - 90°$** and **$45° - 45° - 90°$** triangles. With additional information, you should be able to find the lengths of all sides of one of these special triangles.

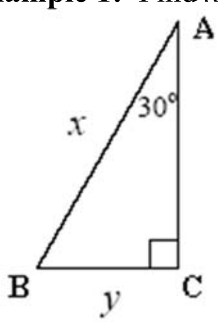
Important Triangles

30-60-90 triangles

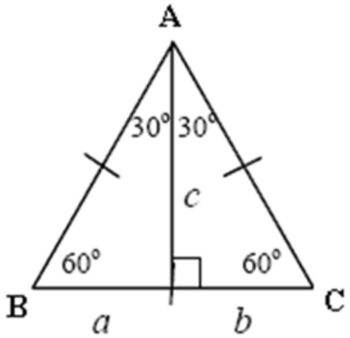
In a $30°$-$60°$-$90°$ triangle, the length of the hypotenuse is two times the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.



Example 1: Find x and y if AC = $4\sqrt{2}$.

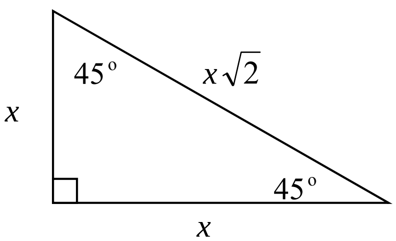


Example 2: In the figure below, an altitude is drawn to the base of equilateral triangle *ABC*. If *AC* = 8, find *a, b* and *c*.



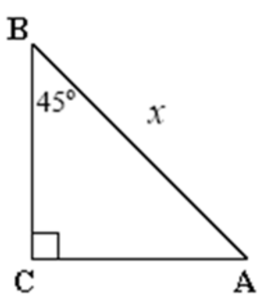
45-45-90 triangles

In a 45°-45°-90° triangle, the legs have the same length. The length of the hypotenuse is $\sqrt{2}$ times the length of either leg.



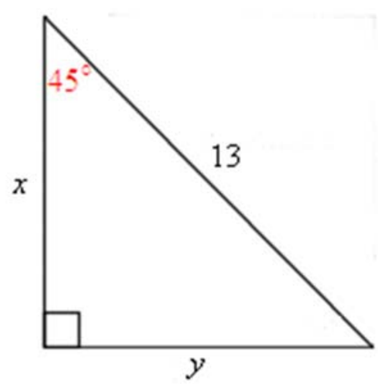
Example 3:

A. Find x if the side AC = $5\sqrt{3}$.



B.

Find x and y.



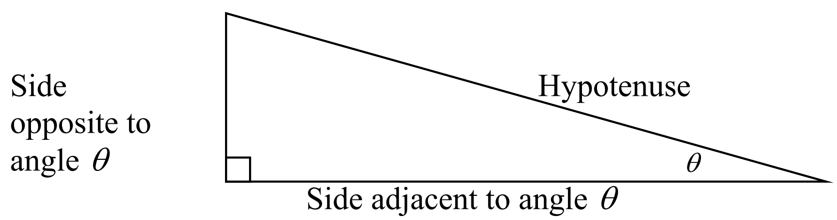
The Six Trigonometric Ratios of an Angle

The word trigonometry comes from two Greek roots, trignon, meaning "having three sides," and meter, meaning "measure." We have already defined the six basic trigonometric functions in terms of a right triangle and the measure of three sides.
A trigonometric function is a ratio of the lengths of the sides of a triangle. If we fix an angle, then as to that angle, there are three sides, the adjacent side, the opposite side, and the hypotenuse. We have six different combinations of these three sides, so there are a total of six trigonometric functions. The inputs for the trigonometric functions are angles and the outputs are real numbers.

The names of the six trigonometric functions, along with their abbreviations, are as follows:

Name of Function	Abbreviation
cosine	cos
sine	sin
tangent	tan
secant	sec
cosecant	csc
cotangent	cot

Let θ be an acute angle places in a right triangle; then



For ease of memorization

A useful mnemonic device:

SOH-CAH-TOA

$$S = \frac{O}{H}$$

$$C = \frac{A}{H}$$

$$T = \frac{O}{A}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle }\theta}{\text{length of hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{\text{length of side opposite to angle }\theta}{\text{length of hypotenuse}} \qquad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{length of side opposite to angle }\theta}{\text{length of side adjacent to angle }\theta} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

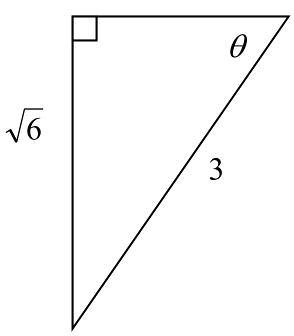
$$\sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle }\theta} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite to angle }\theta} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cot\theta = \frac{\text{length of side adjacent to angle }\theta}{\text{length of side opposite to angle }\theta} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Note: For acute angles the values of the trigonometric functions are always positive since they are ratios of lengths.

Example 4: Find the values of all six trigonometric ratios for the angle θ in the figure below.



Example 5: Suppose a triangle ABC has C = 90º, AC = 7 and AB = 9. Find csc(A) and tan(B).

Example 6: Suppose that θ is an acute angle in a right triangle and $\sec\theta = \frac{5\sqrt{3}}{4}$. Find $\sin\theta$ and $\cos\theta$.