Section 5.2
Graphs of the Sine and Cosine Functions

A Periodic Function and Its Period

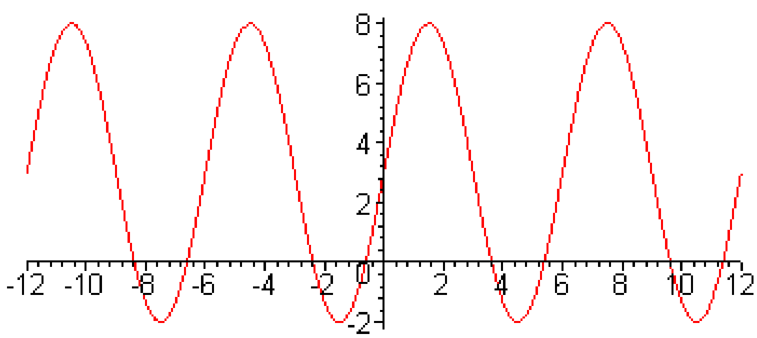
A nonconstant function *f* is said to be **periodic** if there is a number $p > 0$ such that $f(x + p) = f(x)$ for all *x* in the domain of *f*. The smallest such number *p* is called the period of *f*.

The graphs of periodic functions display patterns that repeat themselves at regular intervals.

Amplitude

Let *f* be a periodic function and let *m* and M denote, respectively, the minimum and maximum values of the function. Then the **amplitude** of *f* is the number $\frac{M - m}{2}$.
In other words the amplitude is half the height.

Example 1:
Specify the period and amplitude of the given function.



Now let's talk about the graphs of the sine and cosine functions.

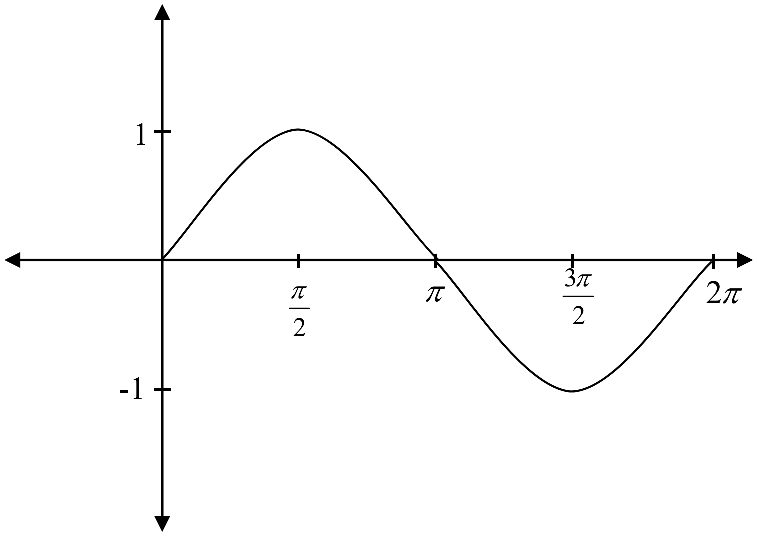
Recall: $\sin(\theta + 2\pi) = \sin\theta$ and $\cos(\theta + 2\pi) = \cos\theta$

This means that after going around the unit circle once (2π radians), both functions repeat. So the period of both sine and cosine is 2π. Hence, we can find the whole number line wrapped around the unit circle.

Since the period of the sine function is 2π, we will graph the function on the interval $[0, 2\pi]$. The rest of the graph is made up of repetitions of this portion.

The previous information leads us to the graphs of sine and cosine…

Sine: $f(x) = \sin x$



Period: 2π

Amplitude: 1

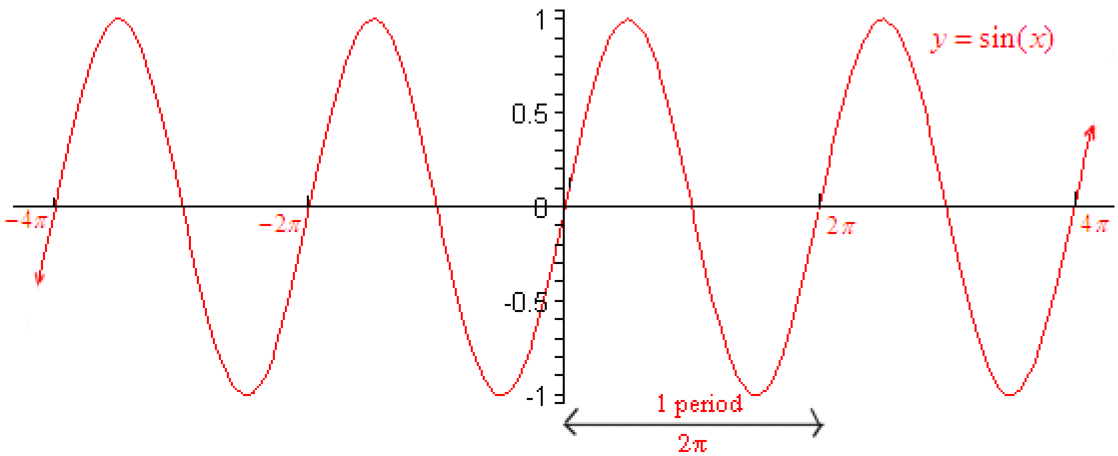
x- intercepts: $k\pi$, k is an integer.

y- intercept: (0, 0)

Domain: $(-\infty, \infty)$

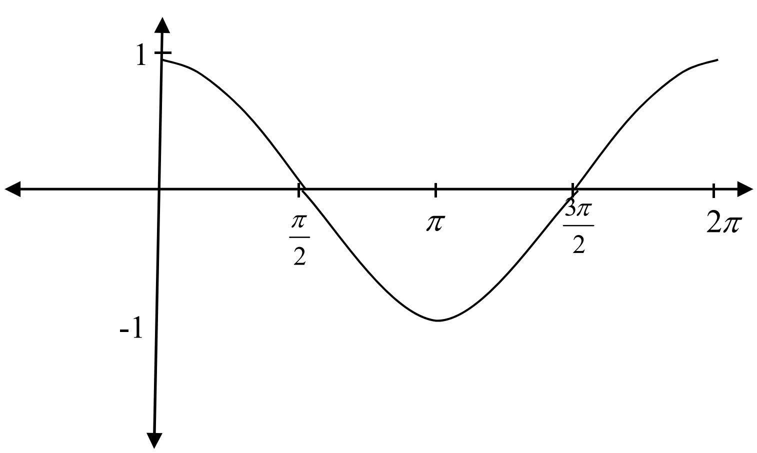
Range: [-1, 1]

Big picture:



Since the period of the cosine function is 2π, we will graph the function on the interval $[0, 2\pi]$. The rest of the graph is made up of repetitions of this portion.

Cosine: $f(x) = \cos x$



Period: 2π

Amplitude: 1

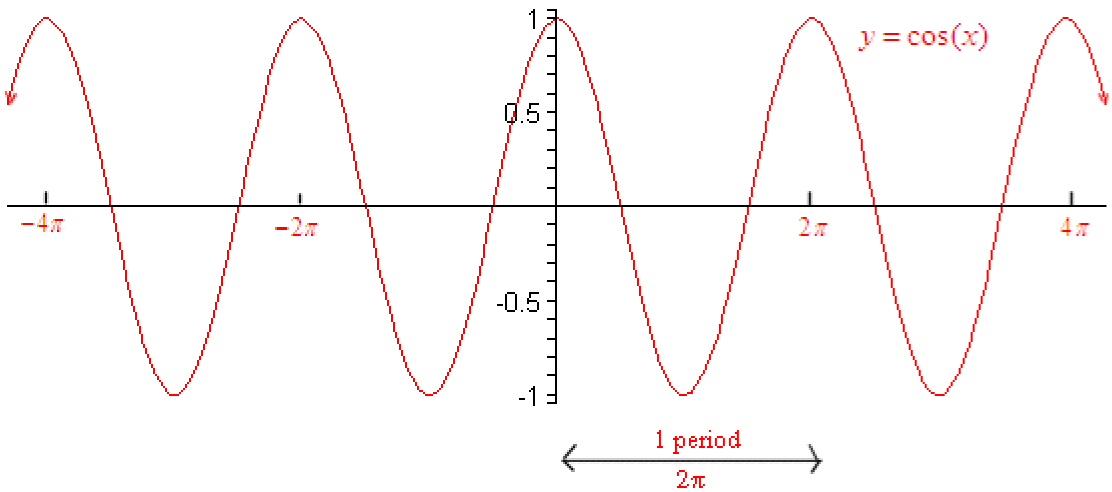
x-intercepts: $\frac{k\pi}{2}$, k is an odd integer

y-intercept: (0, 1)

Domain: $(-\infty, \infty)$

Range: [-1, 1]

Big picture: –



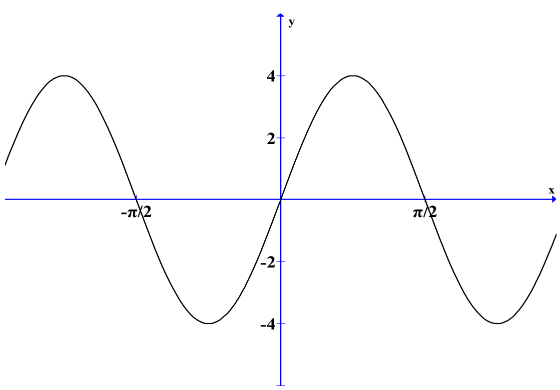
Note: The graphs of $y = \sin x$ and $y = \cos x$ are exactly the same shape. The only difference is that to get the graph of $y = \cos x$, simply shift the graph of $y = \sin x$ to the left $\frac{\pi}{2}$ units. It's a fact that $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$.

For the following functions: $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$

-Amplitude = $|A|$ (Note: Amplitude is always positive.)

-Period = $\frac{2\pi}{B}$

-Translation in horizontal direction (called the *phase shift*) = $\frac{C}{B}$



Note that amplitude vertically stretches or shrinks the graph. So if A is between $0 < 1$ then the graph will vertically shrink. If A is >1 then the graph will stretch vertically. The period horizontally stretches and shrinks the same graph. So if B >1 means the graph will shrink horizontally and if $0 < B < 1$ then the graph will stretch horizontally.

One complete cycle of the sine curve includes three *x*-intercepts, one maximum point and one minimum point. The graph has *x*-intercepts at the beginning, middle, and end of its full period. Key points in graphing sine functions are obtained by dividing the period into four equal parts.

The graph of $y = A\sin(Bx - C)$ completes one cycle from $x = \frac{C}{B}$ to $x = \frac{C}{B} + \frac{2\pi}{B}$.

One complete cycle of the cosine curve includes two *x*-intercepts, two maximum points and one minimum point. The graph has *x*-intercepts at the second and fourth points of its full period. Key points in graphing cosine functions are obtained by dividing the period into four equal parts.

The graph of $y = A\cos(Bx - C)$ completes one cycle from $x = \frac{C}{B}$ to $x = \frac{C}{B} + \frac{2\pi}{B}$.

Example 2: State the transformations for:

a. $f(x) = -2\sin(x+2)+3$

b. $g(x) = \cos\left(2x - \frac{\pi}{4}\right)$

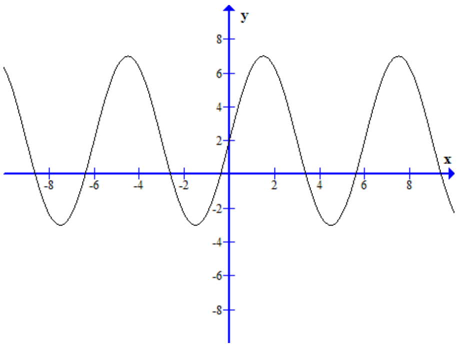
c. $h(x) = \frac{1}{2}\sin\left(\frac{\pi}{4}x\right)$

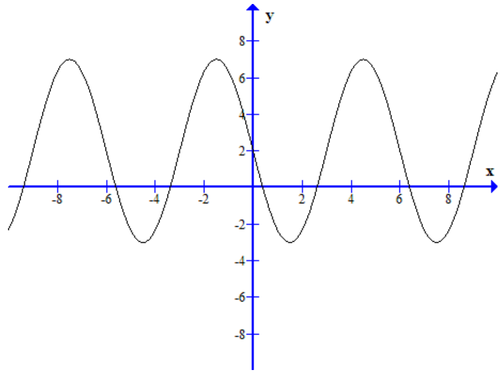
Example 3: Graph $f(x) = 3\sin(2x)$.

Example 4: Graph $f(x)=\sin\left(2x+\frac{\pi}{2}\right)-1$.

Example 5: Graph $-4\cos(5x)$

Example 6: Given the function describe the amplitude, period, phase shift, and vertical shift. Then which graph is correct. $f(x) = 5\sin\left(\frac{\pi}{3}x + \pi\right) + 2$





Example 7: Given the function describe the amplitude, period, phase shift, and vertical shift. Then which graph is correct. $f(x) = 2\cos\left(2x + \frac{\pi}{2}\right)$