Section 5.4b
Inverse Trigonometric Functions and Models

Here is a summary of properties that maybe helpful when evaluating inverse trigonometric functions:

$\sin(\sin^{-1}(x)) = x$ when $x \in [-1, 1]$
$\cos(\cos^{-1}(x)) = x$ when $x \in [-1, 1]$
$\tan(\tan^{-1}(x)) = x$ when $x \in (-\infty, \infty)$

$\sin^{-1}(\sin(x)) = x$ when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}(\cos(x)) = x$ when $x \in [0, \pi]$
$\tan^{-1}(\tan(x)) = x$ when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Example 1: Find the exact value, if possible.

a. Find $\tan^{-1}(-\sqrt{3})$

b. Evaluate**:** $\mathbf{\sin^{-1}(\sin(\boldsymbol{\pi}))}$

c. Evaluate**:** $\mathbf{\cos(\cos^{-1}(\boldsymbol{\pi}))}$

Example 2: Evaluate $\cos\left(\sin^{-1}\left(-\frac{3}{4}\right)\right)$

Example 3: Evaluate $\cot\left(\csc^{-1}\left(\frac{7}{4}\right)\right)$.

Example 4: Find the exact value: $\cos\left[\sin^{-1}\left(\frac{5}{13}\right)\right]$.

Models

As we know, trigonometric functions repeat their behavior. Breathing normally, brain waves during deep sleep are just a couple of examples that can be described using a sine function.

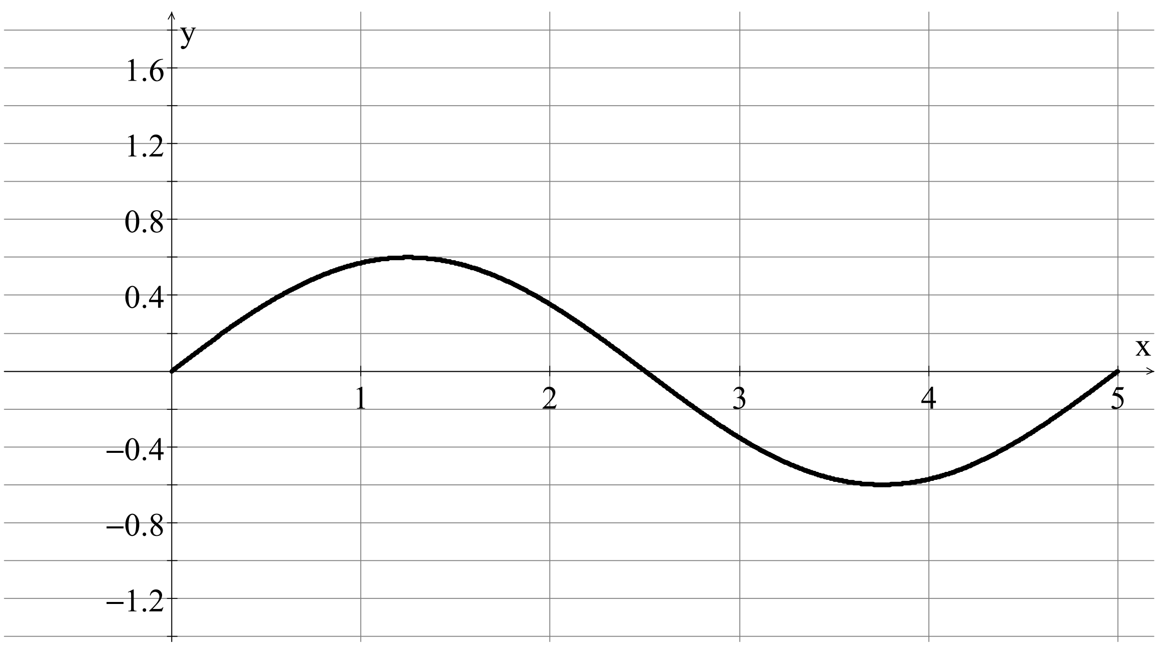
Example 5: Determine the equation of the sine function which has amplitude is 5, the phase shift is 4 to the left, the vertical shift is 3 down, and the period is 2.

Example 6: The number of hours of daylight in Boston is given by $f(x) = 3\sin\frac{2\pi}{365}(x-79)+12$, where x is the number of days after January 1. What is the:

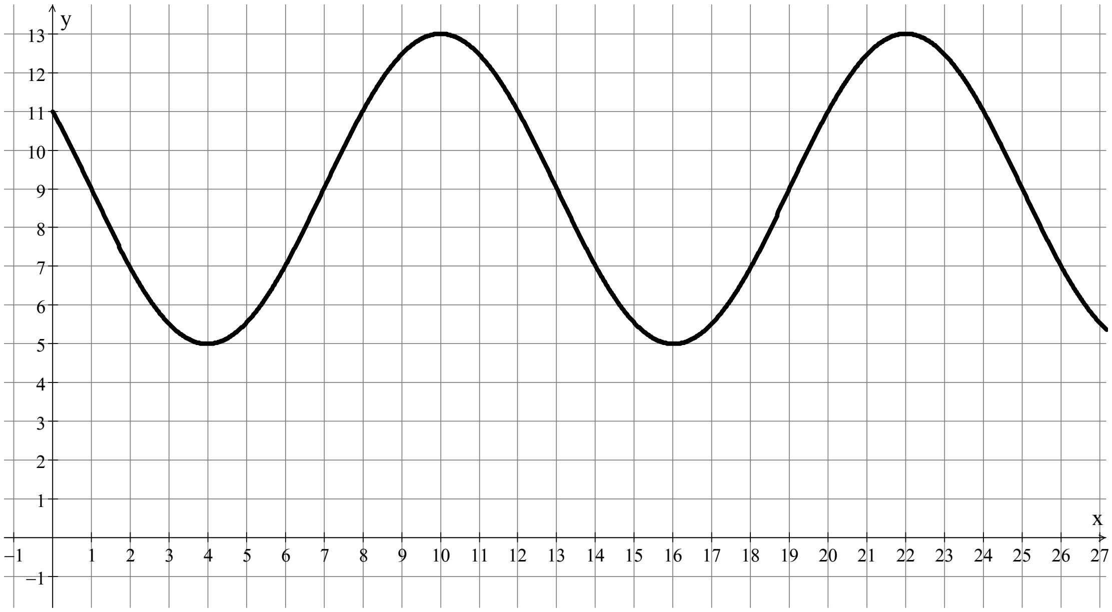
a. amplitude?

b. period?

Example 7: Determine the equation of the form $f(x) = A\sin(Bx)$ for the following graph.



Example 8: Determine the equation of the form $f(x) = A\sin B(x - C/B) + D$ for the following graph.



Example 9: Your fishing bobber oscillates in simple harmonic motion from the waves in the lake where you fish. Your bobber moves a total of 1.5 inches from its high point to its low point and returns to its high point every 3 seconds. Write an equation of the form $f(x) = A\cos(Bx)$ modeling the motion of your bobber, if it is at its high point at time t = 0.

Example 10: Assume you are aboard a submarine. You begin to alternate deeper and then shallower. At time t = 5 min you are at your deepest, y = -900 m. At time t = 10 min you next reach your shallowest, y = -300 m. Assume y varies with time. Find an equation of the form $f(x) = A\cos B(x - C/B) + D$ that describes this situation.