Section 4.4
Trigonometric Expressions and Identities

In this section, you'll learn to simplify trig expressions using identities and using basic algebraic operations. You can add, subtract, multiply, divide and factor trig expressions, in much the same manner than you can with algebraic expressions.

Example 1: Perform the following operation and simplify: $[\sec(\theta)+5][\sec(\theta)-6]$

Example 2: Factor: $\sin^2 x - 4\sin x + 3$

Example 3: Factor: $\cos(\theta)^2 - 1$

Example 4: Simplify: $\frac{3}{\cos\theta} + \frac{5}{\sin\theta}$

Example 5: Factor: $sec^2\theta - 3sec\theta - 10$

Here are some identities you should already know:

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

$$\cot(t) = \frac{\cos(t)}{\sin(t)}$$

Reciprocal Identities

$$\csc(t) = \frac{1}{\sin(t)}, \sin(t) \neq 0$$

$$\sec(t) = \frac{1}{\cos(t)}, \cos(t) \neq 0$$

$$\cot(t) = \frac{1}{\tan(t)}, \tan(t) \neq 0$$

Pythagorean Identities

You should either memorize or be able to derive the other two.

$$\sin^2(t) + \cos^2(t) = 1$$

$$\tan^2(t) + 1 = \sec^2(t)$$

$$1 + \cot^2(t) = \csc^2(t)$$

We can simplify trigonometric expressions by making substitutions using these identities and performing appropriate algebra.

Example 6: Simplify: $\tan(x)\csc(x)$

Example 7: Simplify: $\dfrac{\csc^2(\theta)-1}{1-\sin^2(\theta)}$

Example 8: Simplify: $\dfrac{\sec^2(\theta)}{\tan(\theta)+\cot(\theta)}$

At times, you will be asked to verify identities. To do this, you'll use the identities and algebraic operations to show that the left-hand side of the problem equals the right-hand side of the problem.

Here are some pointers for helping you verify identities:

1. Remember that your task is to show that the two sides of the equation are equal. You may not assume that they are equal.
2. Choose one side of the problem to work with and leave the other one alone. You'll use identities and algebra to convert one side so that it is identical to the side you left alone. You'll work with the "ugly" or more complicated side.
3. If is often helpful to convert all trig functions into sine and cosine. This is usually very helpful! (Unless it makes things worse!!)
4. Find common denominators, if appropriate.
5. Don't try to do too much in one step. Take it one step at a time!
6. If working with one side doesn't get you anywhere, try working with the other side instead.

Example 9: $\dfrac{-1}{\sin^2 x - 1} = \sec^2 x$

Example 10: Prove the identity: $\dfrac{\sin x \cos x}{1 - \cos^2 x} = \cot x$

Example 11: Prove the identity: $\dfrac{\cot A(1 + \tan^2 A)}{\tan A} = \csc^2 A$

Example 12: Prove the identity: $\dfrac{\sin(x)}{1-\cos(x)} = \cot(x) + \csc(x)$

You can also use the identities to help you solve problems like this one. (Note, you can also use a triangle to help you work this problem.)

Example 13: If $\cot(\theta) = \dfrac{-5}{12}, \dfrac{\pi}{2} < \theta < \pi$, find the exact values of the remaining trig functions of θ.