5.1b Start of Identities

Learn these now

Reciprocal Identities

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities:

$$\sin^2(t) + \cos^2(t) = 1$$
$$\tan^2(t) + 1 = \sec^2(t)$$
$$1 + \cot^2(t) = \csc^2(t)$$

Opposite Angle Identities

$$\sin(-t) = -\sin(t)$$
$$\cos(-t) = \cos(t)$$
$$\tan(-t) = -\tan(t)$$
$$\csc(-t) = -\csc(t)$$
$$\sec(-t) = \sec(t)$$
$$\cot(-t) = -\cot(t)$$

Example 1: Simplify: $\cot(-t)\sec(-t)$

Here's another set of identities:

Periodicity

The sine and cosine functions are periodic functions. That means that there is some number p such that $f(x+p)=f(x)$. The number p is the period of the function. So

$\sin(t+2\pi)=\sin(t)$ more generally $\sin(t+2k\pi)=\sin(t)$
$\cos(t+2\pi)=\cos(t)$ $\cos(t+2k\pi)=\cos(t)$

for all real numbers t and all integers k.

The tangent and cotangent functions are also periodic functions. However, these functions repeat themselves when $\mathbf{p}=\pi$. So

$\tan(t+\pi)=\tan(t)$ more generally $\tan(t+k\pi)=\tan(t)$
$\cot(t+\pi)=\cot(t)$ $\cot(t+k\pi)=\cot(t)$

for all real numbers t and all integers k.

Note: the period for the sine and cosine functions is 2π while the period for the tangent and cotangent functions is π.

The secant and cosecant functions are the reciprocal functions , so they will follow the same periodicity rules as sine and cosine.

$\sec(t+2\pi k)=\sec(t)$
$\csc(t+2\pi k)=\csc(t)$ for all real numbers t and all integers k.

Example 2: Simplify: $\dfrac{1+\tan(t-\pi)}{1+\cot(t+2\pi)}$

Example 3: Suppose that $\csc(x) = \frac{4}{3}$ and that $0 < x < \frac{\pi}{2}$. Compute $\cot(x - 74\pi)$.

Example 4: Simplify.

$$\frac{\sin(t + 6\pi)\csc(t - 2\pi)}{\cot(t + \pi) + \tan(t + 2\pi)}$$

Example 5: Find the equivalent: $\frac{\sec^2 x - 1}{\sec^2 x}$.

Example 6: Find the equivalent: $\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}$