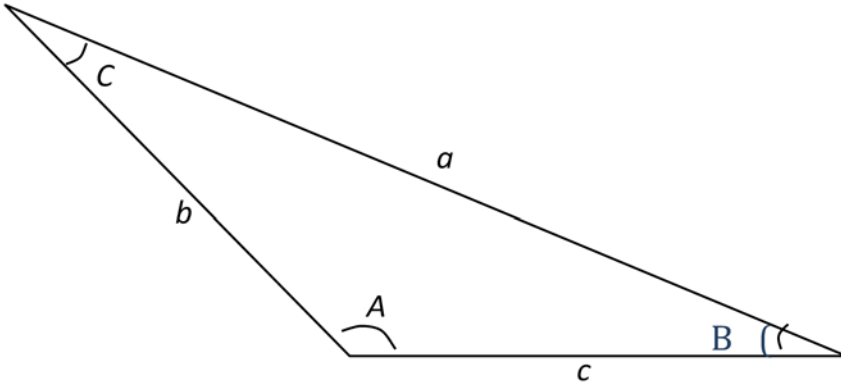**Section 7.3**   **Law of Sines and Law of Cosines**

a,b,c are lengths of sides. A, B, C are angles (A is opposite a, etc.)

Law of Sines: *The sines of angles are proportional to the lengths of opposite sides.*

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Used for *SAA*, *SSA* cases.

<u>*Law of Cosines*</u>**:** Generalized Pythagorean theorem.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Used for *SAS*, *SSS* cases.

We use these laws to find angles and side lengths for triangles of any type (not just right triangles).

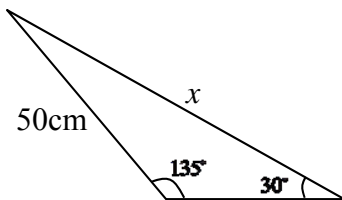| *Notation*	*Given Information*
SAA	One side and two angles
SSA	Two sides and an angle opposite one of the given sides
SAS	Two sides and the included angle
SSS	Three sides

Notice: For a right triangle (with $C = 90°$), we have the following:

$$c^2 = a^2 + b^2 - 2ab\cos(90°)$$
$$c^2 = a^2 + b^2 - 2ab(0)$$
$$c^2 = a^2 + b^2$$

Example 1: Find x.

Note: *SSA* case is called the ambiguous case of the law of sines. There may be two solutions, one solution, or no solutions.   You should throw out the results that don't make sense. That is, if $\sin A > 1$ or the angles add up to more than $180°$.

Example 2: Find all possible measures for the indicated angle of the triangle.
ΔDEF, d = 25 mm, e = 13 mm, ∠E = 21°. Find ∠D.

**Example 3:** For triangle *ABC*, with $a = 2$ft, $b = 10$ft, $A = 30°$, find the length of the other side and the measure of the remaining angles.

Example 4: Given $\triangle$ PEZ, p = 6 cm, e = 13 cm, and z = 11 cm. Find $\angle$Z.

Example 5: Given $\triangle$ RUN, r = 7 cm, u = 12 cm, and n = 4 cm. Find $\angle$U.

Example 6: Two sailboats leave the same dock together traveling on courses that have an angle of 135° between them. If each sailboat has traveled 3 miles, how far apart are the sailboats from each other?

Example 7: Determine the angle x in the triangle given below with AB = 8 and BC = 11.
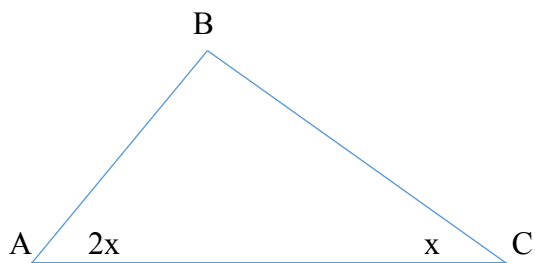
Example 8: Given triangle ABC, the measure of angle A is 60°, the length of BC is $\sqrt{3}$, and the length of AC is the $\sqrt{2}$. How many solutions are there for the measure of angle B?

Example 9: Given triangle ABC, the measure of angle A is 45° , the length of AB is 6, and the length of AC is 6. What is the length of side BC?