Section 6.2
Double-Angle and Half-Angle Formulas

The following, most useful, basic identities follow from the addition formulas.

Double-Angle Formulas	Half-Angle Formulas
$\sin(2\theta) = 2\sin\theta\cos\theta$	$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$	$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$	$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$

Note: In the half-angle formulas the $\pm$ symbol is intended to mean either positive or negative but not both, and the sign before the radical is determined by the quadrant in which the angle $\frac{\theta}{2}$ terminates.

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Example 1: Simplify.

a. $\cos^2\left(\frac{3\pi}{8}\right) - \sin^2\left(\frac{3\pi}{8}\right)$

b. $2\sin(15^\circ)\cos(15^\circ)$

Example 2: True or False: $\tan\left[2\left(\frac{\pi}{4}\right)\right] = 2\tan\left(\frac{\pi}{4}\right)$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$$

Note: In the half-angle formulas the $\pm$ symbol is intended to mean either positive or negative but not both, and the sign before the radical is determined by the quadrant in which the angle $\frac{\theta}{2}$ terminates.

Example 3: Use the half-angle formula to calculate $\cos\left(\frac{\pi}{8}\right)$. *Recall:* $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

a. Which quadrant does $\frac{\pi}{8}$ live in?

b. Is $\cos\left(\frac{\pi}{8}\right)$ positive or negative?

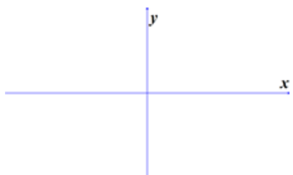
c. Rewrite $\cos\left(\frac{\pi}{8}\right)$ so that it's in the form $\cos\left(\frac{\theta}{2}\right)$ and then calculate.

What does θ equal?

Example 4: Given $\sin\left(\frac{13\pi}{8}\right)$.

a. Which quadrant does $\frac{13\pi}{8}$ live in?

b. Is $\sin\left(\frac{13\pi}{8}\right)$ positive or negative?



c. If $\sin\left(\frac{13\pi}{8}\right)$ is rewritten in the form $\sin\left(\frac{\theta}{2}\right)$ what would θ equal?

Example 5: Given $\cos\left(\frac{7\pi}{12}\right)$.

a. Which quadrant does $\frac{7\pi}{12}$ live in?

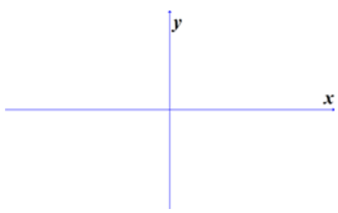
b. Is $\cos\left(\frac{7\pi}{12}\right)$ positive or negative?

c. If $\cos\left(\frac{7\pi}{12}\right)$ is rewritten in the form $\cos\left(\frac{\theta}{2}\right)$ what would θ equal?

Example 6: Use the half-angle formula to calculate $\sin\left(\frac{13\pi}{12}\right)$. *Recall:* $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

a. Which quadrant does $\frac{13\pi}{12}$ live in?

b. Is $\sin\left(\frac{13\pi}{12}\right)$ positive or negative?



c. Rewrite $\sin\left(\frac{13\pi}{12}\right)$ so that it's in the form $\sin\left(\frac{\theta}{2}\right)$ and then calculate.

What does θ equal?

d. Use the sum and difference formula to calculate $\sin\left(\frac{13\pi}{12}\right)$.

Example 7: Suppose $\sin\theta = -\frac{4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$.

a. Find $\sin(2\theta)$.

b. Find $\cos\left(\frac{\theta}{2}\right)$.

- When calculating trigonometric functions of multiples of $\frac{\pi}{12}$, you have the choice of using an addition formula or using a half-angle formula.

- When calculating trigonometric functions of multiples of $\frac{\pi}{8}$, you have only one choice: a half-angle formula.

  It is not possible to write $\frac{\pi}{8}$ as a sum or difference of our special angles $\frac{\pi}{3}, \frac{\pi}{6}$, and $\frac{\pi}{4}$!!!