

MATH 2311

Section 2.1

Counting Techniques

Combinatorics is the study of the number of ways a set of objects can be arranged, combined, or chosen; or the number of ways a succession of events can occur. Each result is called an **outcome**. An **event** is a subset of outcomes. When several events occur together, we have a **compound event**.

Outcome: all ways that something can happen.

Event: specific conditions about how something can happen.

The Fundamental Counting Principle

The **Fundamental Counting Principle** states that the total number of ways a compound event may occur is $n_1 \cdot n_2 \cdot n_3 \cdot \dots \cdot n_i$ where n_1 represents the number of ways the first event may occur, n_2 represents the number of ways the second event may occur, and so on.

Example:

You are not allowed repeats (such as "double mushrooms")

Example:

How many ways can you create a pizza choosing a meat and two veggies if you have 3 choices of meats and 4 choices for veggies?

$$\frac{3}{\text{Meat}} \times \frac{4}{\text{Veg 1}} \times \frac{3}{\text{Veg 2}} = 36 \text{ options}$$

(one of the four was used in step, 3 options remain)

Rstudio and TI Commands:

Rstudio:

factorials: (such as 5!)

factorial(5)

combinations: (such as ${}_6C_2$)

choose(6,2)

TI 83/84:

Select MATH → PRB (right arrow)

factorial: (option 4)

5!

permutation: (option 2)

${}_6P_2$ will be written as 6 nPr 2

combination: (option 3)

${}_6C_2$ will be written as 6 nCr 2

(No permutation for Rstudio)

Permutations

A **permutation** of a set of n objects is an ordered arrangement of the objects.

$${}_n P_n = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1 = \underline{n!}$$

All objects are placed in order

$${}_n P_r = \frac{n!}{(n-r)!}$$

Some of the objects are placed in order

Examples:

How many ways can six people be seated in a row? All six are being seated (factorial)

$6!$

```
> factorial(6)
[1] 720
```

```
6!
720
```

720

! =

In how many ways can 3 of the six symbols, &^%\$#@ be arranged?

Since we are only arranging 3 out of the 6 options, this is a permutation.

$6P_3$

```
6 nPr 3
120
```

P(,) =

120

With Repetition

When we allow repeated values, The number of orderings of n objects taken r at a time, with repetition is n^r . Example:

n : number of items we are selecting from

r : number of selections being made

In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

$$n = 6$$
$$r = 4$$

$$6^4 = 1296$$
$$(n^r)$$

Duplicate Objects

The number of permutations, P , of n objects taken n at a time with r objects alike, s of another kind alike, and t of another kind alike is

$$P = \frac{n!}{r!s!t!}$$

$r, s, t, \text{ etc}$ are the individual categories
 n is the total number of items to select from
($r+s+t+\dots=n$)

Example:

How many different words (they do not have to be real words) can be formed from the letters in the word MISSISSIPPI?

$$P = \frac{n!}{r!s!t!}$$

$$\begin{array}{l} M: 1 \\ I: 4 \\ S: 4 \\ P: 2 \\ \hline \text{Total: } 11 \end{array}$$

$$P = \frac{11!}{\cancel{(1!)}(4!)(4!)(2!)}$$

Note: $1! = 1$

```
> factorial(11)/(factorial(4)*factorial(4)*factorial(2))  
[1] 34650
```

Circular Permutations

The number of circular permutations of n objects is $(n - 1)!$

Example:

In how many ways can 12 people be seated around a circular table?

$$(12 - 1)!$$

```
> factorial(12-1)
[1] 39916800
> factorial(11)
[1] 39916800
```

Combinations:

Order does not matter

Treatment of each selection is the same.

A **combination** gives the number of ways of picking r unordered outcomes from n possibilities. The number of combinations of a set of n objects taken r at a time is

$${}_n C_r = \frac{n!}{r!(n-r)!}$$

Creating a smaller group from a larger group: combination

Example:

How many ways can a committee of 5 be chosen from a group of 12 people?

$12 C 5$

```
· choose(12,5)
```

```
[1] 792
```

```
12 nCr 5  
792
```

792

C(,) =

Permutation or Combination:

How many ways can 1st, 2nd, and 3rd place ribbons be awarded when there are 15 contestants? Different treatment of selections: permutation

$$P(15, 3) = 2730$$

How many ways can you be dealt a poker hand of 7 cards from a standard deck of 52? No special treatment for different selections: combination

$$C(52, 7) = 133784560$$

How many ways can a class President, Vice President, Secretary, Treasurer, and Historian be selected from a class of 500?

Each selection has a different treatment: permutation

$$P(500, 5) = 30629362512000$$