

# MATH 1342

## Section 2.3

# Basic Probability Models

- The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.  
This is a measure of how likely an event is to occur.  
Example: How likely is it to roll a 4 on a 6-sided die
- The **sample space** of a random phenomenon is the set of all possible outcomes.  
All possible numbers that can turn up:  
 $S = \{1, 2, 3, 4, 5, 6\}$
- An **event** is an outcome or a set of outcomes of a random phenomenon. It is a subset of the sample space. A **simple event** is an event consisting of exactly one outcome.  
The desired results (subset of the sample space)  
 $E = \{4\}$

# Computing Probability

- To compute the probability of some event  $E$  occurring, divide the number of ways that  $E$  can occur by the number of possible outcomes the sample space,  $S$ , can occur:

$$P(E) = \frac{n(E)}{n(S)}$$

(number of elements in event)/  
(number of elements in sample)

$P(4) = 1/6$

# Basic Rules of Probability

0: Probability of something impossible

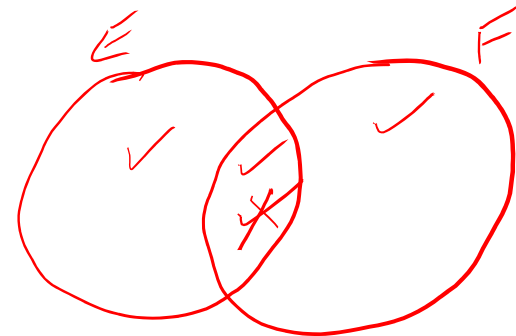
1: Probability of something guaranteed

1. All events have a probability between zero and one.  $0 \leq P(E) \leq 1$
2. All possible outcomes together must have a probability of one.  $P(S) = 1$
3. Complement Rule: For any event  $E$ ,  $P(E^c) = 1 - P(E)$
4. Addition Rule: If A and B are disjoint events, then  $P(E \cup F) = P(E) + P(F)$
5. If  $E$  and  $F$  are **any** events of an experiment, then  $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

#3:  $P(\text{Not } 4) = 1 - P(4) = 1 - 1/6 = 5/6$

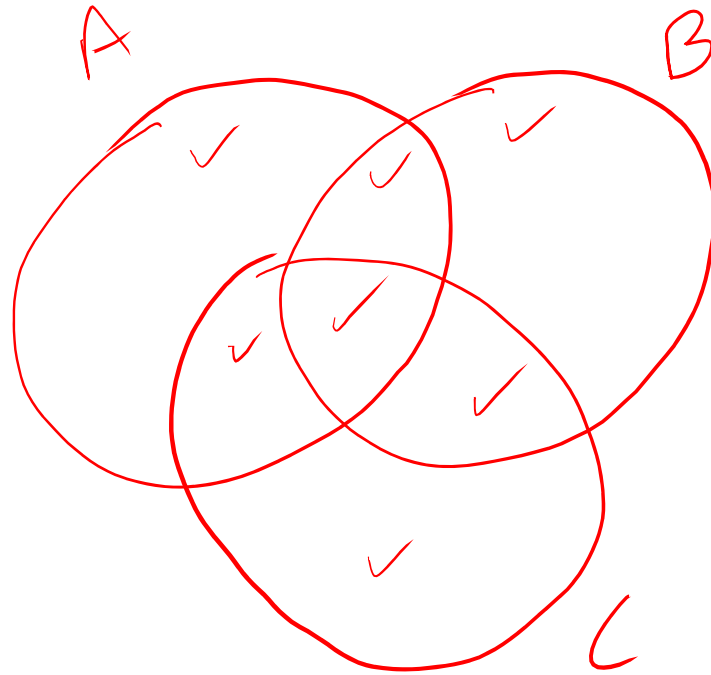
#4:  $P(4 \text{ or } 5) = P(4) + P(5) = 1/6 + 1/6 = 2/6 = 1/3$

#5:  $P(4 \text{ or even}) = P(4) + P(\text{even}) - P(4 \text{ and even}) = 1/6 + 3/6 - 1/6 = 3/6 = 1/2$



Rule 5a:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$



# A deck of cards:

52 total cards

26 red cards

13 hearts : A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

13 diamonds: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

26 black cards

13 spades: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

13 clubs: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Popper 02:

a. 0      b. 0.5      c. 0.538      d. 0.75      e. 1

You are selecting one card from a standard deck of 52 cards:

1. What is the probability that the card is red or a club?

Rule #4:  $P(\text{Red or Club}) = P(\text{Red}) + P(\text{Club}) = 26/52 + 13/52 = 0.75$       Choice D

2. What is the probability that the card is a queen or a black card?

Rule #5:  $P(\text{Queen or Black}) = P(Q) + P(B) - P(Q \text{ and } B) = 4/52 + 26/52 - 2/52 = 0.538$       Choice C

3. What is the probability that the card is an emperor?

Rule #1:  $P(E) = 0$  *There is no emperor card in a deck of cards*      Choice A

4. What is the probability that the card is not a spade?

Rule #3:  $P(\text{Not } S) = 1 - P(S) = 1 - 13/52 = 0.75$       Choice D

5. What is the probability that the card is a diamond, heart, spade or club?

Rule #2:  $P(D \text{ or } H \text{ or } S \text{ or } C) = P(\text{Sample}) = 1$       Choice E

## Examples:

If 5 marbles are drawn at random all at once from a bag containing 8 white and 6 black marbles, what is the probability that 2 will be white and 3 will be black?

$$\begin{array}{r} \text{Sample} \\ \text{Black: } 6 \\ \text{white: } 8 \\ \hline \text{Total: } 14 \end{array}$$

$$\begin{array}{r} \text{Event} \\ \text{Black: } 3 \\ \text{white: } 2 \\ \hline \text{Total: } 5 \end{array}$$

$$\begin{array}{r} \text{Black} \quad \text{white} \\ 6C_3 \times 8C_2 \\ \hline 14C_5 \\ \text{Total} \end{array}$$

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> (choose(6,3)*choose(8,2))/choose(14,5)
[1] 0.2797203
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Examples:

$$\begin{array}{r} \text{Sample} \\ \text{Men: } 5 \\ \text{Women: } 7 \\ \hline \text{Total: } 12 \end{array}$$

$$\begin{array}{r} \text{Event (a)} \\ \text{Men: } 0 \\ \text{Women: } 6 \\ \hline \text{Total: } 6 \end{array}$$

$$\begin{array}{r} \text{Event (b)} \\ \text{Men: } 3 \\ \text{Women: } 3 \\ \hline \text{Total: } 6 \end{array}$$

The qualified applicant pool for six management trainee positions consists of seven women and five men.

- a. What is the probability that a randomly selected trainee class will consist entirely of women?

$$\frac{{}^5C_0 \times {}^7C_6}{{}^{12}C_6} > \text{choose}(5,0) * \text{choose}(7,6) / \text{choose}(12,6)$$

[1] 0.007575758

- b. What is the probability that a randomly selected trainee class will consist of an equal number of men and women?

$$\frac{{}^5C_3 \times {}^7C_3}{{}^{12}C_6}$$

$$> \text{choose}(5,3) * \text{choose}(7,3) / \text{choose}(12,6)$$

[1] 0.3787879

## Examples:

A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.

a. What is the probability that a randomly selected student likes basketball or hockey? Solve this by also using an appropriate formula.

$$\begin{aligned}P(B \text{ or } H) &= P(B) + P(H) - P(B \text{ and } H) \\ &= .66 + .38 - .22 = .82\end{aligned}$$

## Examples:

A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.

b. What is the probability that a randomly selected student does not like any of these sports?

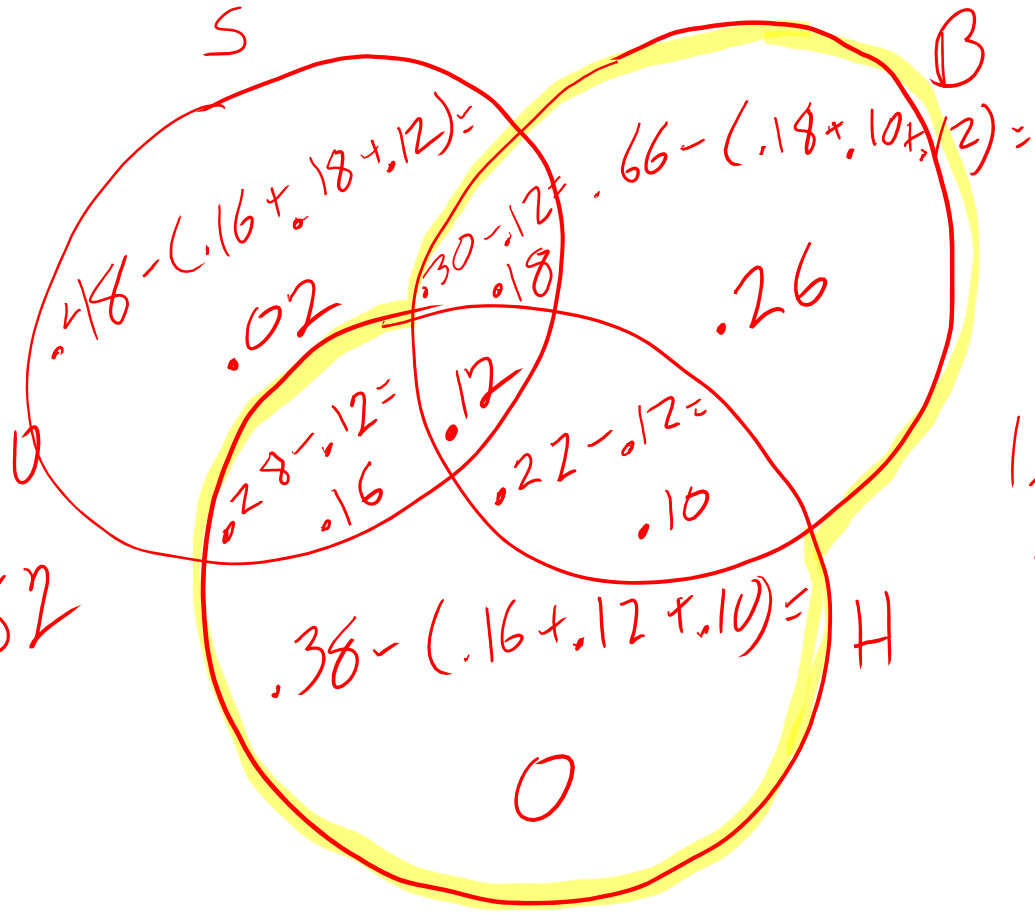
$$1 - P(S \cup B \cup H) = 1 - .84 = \boxed{.16}$$

$$\begin{aligned} P(S \cup B \cup H) &= P(S) + P(B) + P(H) - P(S \cap B) - P(S \cap H) - \\ &\quad P(B \cap H) + P(S \cap B \cap H) \\ &= .48 + .66 + .38 - .30 - .22 - .28 + .12 = .84 \end{aligned}$$

A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.

Question 1:

$$.16 + .12 + .26 + .10 + .18 + 0 = .82$$



$$1.00 - (.02 + .18 + .26 + .16 + .12 + .10 + 0) =$$

.16  
 ↑ Question 2