MATH 2311

Section 2.4

General Probability Rules

Two events are **independent** if knowing that one occurs does not change the probability that the other occurs.

(Note: This is not the same as sets that are disjoint or mutually exclusive)

If E and F are independent events, then $P(E \cap F) = P(E)P(F)$

This is also used as a test of independence

Example:

If P(A) = .36 and P(B) = .58 and A and B are independent, what is P(A and B)?

$P(A \cap B) = P(A) \cdot P(B) = (.36) \cdot (.58) = .2088$

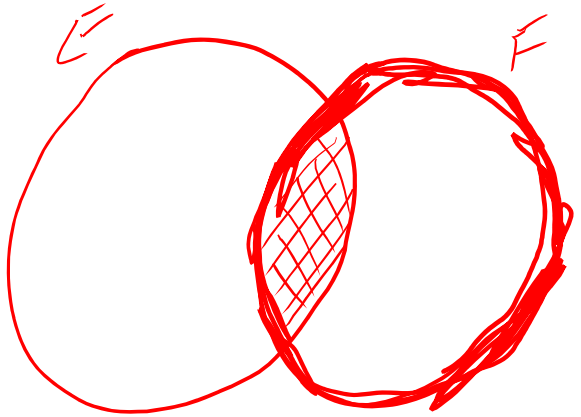
Conditional Probability

Dependent Events

Dependent events, the occurrence of one event does have an effect on the occurrence of the other event. The probability $P(E \mid F)$ is read "the probability of event E given event F had already occurred". If E and F are independent, then $P(E \mid F) = P(E)$. If events E and F are dependent then $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$.

This means $P(E \cap F) =$ $P(E \mid F) \cdot P(F)$

(Cross multiplication)



Examples:

A clothing store that targets young customers (ages 18 through 22) wishes to determine whether the size of the purchase is related to the method of payment. A sample of 300 customers was analyzed and the information is below:

	Cash	Credit	Layaway	Total
Under $40	60	30	10	100
$40 or more	40	100	60	200
Total	100	130	70	300

	Cash	Credit	Layaway	Total
Under $40	60	30	10	100
$40 or more	40	100	60	200
Total	100	130	70	300

a. If a customer is selected at random from this group of customers, what is the probability that the customer paid cash?

$P(\text{Cash}) = \frac{100}{300} = 0.333$

b. If a customer is selected at random from this group of customers, what is the probability that the customer paid with a credit card?

$P(\text{Credit}) = \frac{130}{300} = .433$

c. If a customer is selected at random from this group of customers, what is the probability that the customer paid with the layaway plan?

$P(\text{Layaway}) = \frac{70}{300} = 0.233$

	Cash	Credit	Layaway	Total
Under $40	60	30	10	100
$40 or more	40	100	60	200
Total	100	130	70	300

d. If a customer is selected at random from this group of customers, what is the probability that the customer purchased under $40?

$$P(<40) = \frac{100}{300} = .333$$

e. If a customer is selected at random from this group of customers, what is the probability that the customer purchased $40 or more?

$$P(\geq 40) = \frac{200}{300} = .667$$

	Cash	Credit	Layaway	Total
Under $40	60	30	10	100
$40 or more	40	100	60	200
Total	100	130	70	300

P(Credit) = .433

P(LA) = .233

f. If a customer is selected at random from this group of customers, what is the probability that the customer paid with a credit card given that the purchase was under $40?

$$P(\text{Credit} \mid <40) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)} = \frac{30}{100} = 0.3$$

$\neq P(\text{Credit})$

Dependent

g. If a customer is selected at random from this group of customers, what is the probability that the customer paid with the layaway plan given that the purchase was $40 or more?

$$P(LA \mid \geq 40) = \frac{n(A \cap B)}{n(B)} = \frac{60}{200} = 0.3 \neq P(LA)$$

Popper 03: P(A) = 0.6 P(B) = 0.3 P(A∪B) = 0.8

1. Determine the value of P(A∩B) = .10 [A]
2. Determine the value of P(A|B) = $\frac{P(A \cap B)}{P(B)}$ $\frac{.10}{.3} = .333$ [C]
3. Determine the value of P(B|A) = $\frac{P(A \cap B)}{P(A)}$ $\frac{.10}{.6} = .167$ [B]

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.8 = .6 + .3 - x$

$.8 = .9 - x$

$-.1 = -x \quad x = .10$

Answer choices for Questions 1, 2, 3:

a. 0.1 b. 0.167 c. 0.333 d. 0.375 e. 0.75

$.6 \cdot .3 = .10 \rightarrow .18 \neq .10$ $.6 \neq .333$ $.3 \neq .167$

4. Which of the following cannot be used to test for independence?

a. P(A) · P(B) = P(A∩B) b. P(A) + P(B) = P(A∪B) c. P(A) = P(A|B) d. P(B) = P(B|A)

5. Are events A and B independent? a. Yes, they are independent b. No, they are dependent

Examples:

$P(D) = .30$ $P(C|D) = .40$ $P(C|D^c) = .60$

$P(D^c) = 1 - .30 = .70$ $P(C^c|D) = 1 - .40 = .60$ $P(C^c|D^c) = 1 - .60 = .40$

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those "felony" students, 40% go on to college. Of the ones who do not face a disciplinary action, 60% go on to coll

a. What is the probability that a randomly selected student both faced a disciplinary action and went on to college?

$P(C \cap D) = P(C|D) \cdot P(D) = .40 \cdot .30 = .12$

b. What percent of the students from the high school go on to college?

$P(C \cap D^c) = P(C|D^c) \cdot P(D^c) = .60 \cdot .70 = .42$

$P(C) = P(C \cap D) + P(C \cap D^c) = .12 + .42 = .54$

c. Show if events {faced disciplinary action} and {went to college} are independent or not.

$P(C) = P(C|D) \rightarrow .54 \neq .40$

Not independent