MATH 1342

Section 3.1

Random Variables

A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.

A **discrete random variable** is one that can assume a countable number of possible values. A **continuous random variable** can assume any value in an interval on the number line.

A **probability distribution table of *X*** consists of all possible values of a discrete random variable with their corresponding probabilities.

Example:

Suppose a family has 3 children. Show all possible gender combinations:

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BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Example:

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

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Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

We need to know the total possible outcomes. 8 outcomes

We need to categorize them by number of girls. 0, 1, 2, 3

We need a probability of each outcome. Next slide

Example:

BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

0 girls: 1/8

1 girl: 3/8

2 girls: 3/8

3 girls: 1/8

x	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8

The sum of the p-values must equal 1

(1/8) + (3/8) + (3/8) + (1/8) = 8/8 = 1

Example:

X	0	1	2	3
$P(X)$	$1/8$	$3/8$	$3/8$	$1/8$

Find the following probabilities:

Find $P(X > 2) = P(X=3)$

$1/8$

$P(X < 1) = P(X=0)$

$1/8$

$P(1 < X \leq 3)$

0 girls:

1 girl:

$P(1 < X \leq 3) = P(X=2) + P(X=3)$

2 girls:

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

3 girls:

Example: Popper # 04

Another example: Suppose you are given the following distribution table:

X	1	2	3	4	5	6	7
P(X)	**0.15**	**0.05**	**0.10**	**?**	**0.10**	**0.15**	**0.15**

,30

Find the following:

1. $P(X = 4)$
   a. 0.10
   b. 0.15
   c. 0.30
   d. 0.05

2. $P(X < 2)$ = $P(X=1)$
   a. 0.10
   b. 0.15
   c. 0.20
   d. 0.05

3. $P(2 < X \leq 5)$
   a. 0.50
   b. 0.15
   c. 0.20
   d. 0.25

   $P(X=3)+P(X=4)+P(X=5)$

   $.10 + .30 + .10$

4. $P(X > 3)$ =
   a. 0.50
   b. 0.75
   c. 0.80
   d. 0.70

$P(X=4)$	.30
$P(X=5)$	.10
$P(X=6)$	.15
$P(X=7)$	.15
	.70

```
> 1-(.15+.05+.10+.10+.15+.15)
[1] 0.3
```

Expected Value:

The mean, or **expected value**, of a random variable *X* is found with the following formula:

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

Multiply each x-value by its corresponding p-value and add those products

What is the expected number of girls in the family above?

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

x	p
0	1/8
1	3/8
2	3/8
3	1/8

$$E[X] = 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8)$$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$$

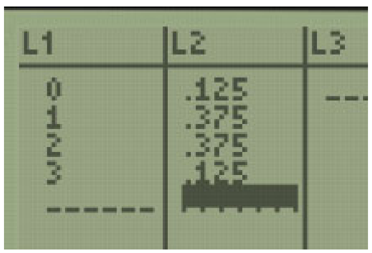
What is the expected number of girls in the family above?

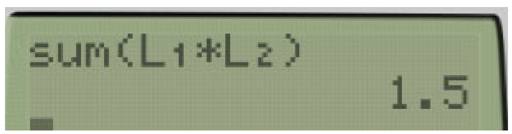
$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

In R Studio: assign("x",c(values))
assign("p",c(probabilities))
sum(x*p)

TI: x → L1, p → L2,
2nd List (STAT), MATH (right arrow), sum (option 5)
sum(L1*L2)

```
> assign("x",c(0,1,2,3))
> assign("p",c(1/8,3/8,3/8,1/8))
> sum(x*p)
[1] 1.5
```





Variance and Standard Deviation

$$\sigma_X^2 = Var[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n$$

$$= \sum (x_i - \mu_X)^2 p_i$$

Or (the alternate formula)

$$\sigma_X^2 = Var[X] = E[X^2] - (E[X])^2$$

Repeat the Expectancy Formula using x^2 instead of x.

Variance and Standard Deviation

$$\sigma_X^2 = Var[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n$$

$$= \sum (x_i - \mu_X)^2 p_i$$

In R Studio: sum((x-mean)^2*p)

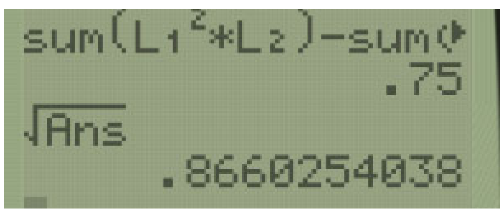
or

sum(x^2*p)-sum(x*p)^2

In TI: sum(L1^2*L2)-sum(L1*L2)^2

Find the **standard deviation** for the number of girls in the example above

```
> sum(x^2*p)-sum(x*p)^2
[1] 0.75
> sqrt(0.75)
[1] 0.8660254
```



Popper 04 continued:

X	1	2	3	4	5	6	7
$P(X)$	0.15	0.05	0.10	?	0.10	0.15	0.15

.30

5\. What is the expected value?

(Mean)

a. 3.50
b. 4.20
c. 0.35
d. 5.00

6, 7. The variance (2) and standard deviation (?)?

6 7

a. 3.66 #6
b. 13.39
c. 1.91 #7
d. 1.55

```
> assign("x",c(1,2,3,4,5,6,7))
> assign("p",c(.15,.05,.10,.30,.10,.15,.15))
> sum(x*p)
[1] 4.2
> sum(x^2*p)-sum(x*p)^2
[1] 3.66
> sqrt(3.66)
[1] 1.913113
```

A carnival game costs \$10 to play. First prize (probability of 0.02) is \$100. Second prize (probability of 0.05) is \$50. Third prize (probability of 0.10) is \$1. What are your expected winnings?

(100 - 10)

x	90	40	-9	-10
p	.02	.05	.10	.83

$\underbrace{.02 \quad .05 \quad .10}_{.17}$

```
> assign("x",c(90,40,-9,-10))
> assign("p",c(.02,.05,.10,.83))
> sum(x*p)
[1] -5.4
```

Rules for means and variances:

Suppose *X* is a random variable and we define *W* as a new random variable such that *W* = *aX* + *b*, where *a* and *b* are real numbers. We can find the mean and variance of *W* with the following formula:

$$E[W] = E[aX + b] = aE[X] + b$$

$$\sigma_W^2 = Var[W] = Var[aX + b] = a^2 Var[X]$$

$$\sigma_W = a[\sigma_x]$$

only use multiplication portion of formula

Rules for means and variances:

Likewise, we have a formula for random variables that are combinations of two or more other independent random variables. Let *X* and *Y* be independent random variables,

$$E[X+Y]=E[X]+E[Y]$$

$$\sigma^2_{X+Y}=Var[X+Y]=Var[X]+Var[Y]$$

and

$$E[X-Y]=E[X-Y]=E[X]-E[Y]$$

$$\sigma^2_{X-Y}=Var[X-Y]=Var[X]+Var[Y]$$

Always add the variances when you are adding or subtracting random variables

Popper 04, continued:

$\mu_x = 22$
$\sigma_x = 3$

Suppose you have a distribution, *X*, with mean = 22 and standard deviation = 3. Define a new random variable *Y* = 3*X* + 1.

8, Find the variance of X. $(\sigma^2 = 3^2 = 9)$ (choice D)

9, Find the mean of Y. $Y = 3(22)+1 = 66+1 = 67$ [choice B]

10, Find the variance of Y. $(\sigma^2 = 9^2 = 81)$ [choice E]

11. Find the standard deviation of Y. $Y = 3(3) = 9$ [choice D]
multiply only

Choices for above questions:

a. 3 b. 67 c. 63 d. 9 e. 81

Use the following Probability Distribution Table to find the values of A, B and C.

X	0	1	2	3	4	5
P(X)	0.15	0.25	0.05	A	B	C

Additional Information: $P(X < 4) = 0.55$; $E[X] = 2.7$

$A = 0.10$
$B = 0.20$
$C = .25$

$P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0.55$

$P(X<4) = .15 + .25 + .05 + A = 0.55$

$.45 + A = 0.55$

$A = 0.10$

$E[X] = \cancel{0(.15)} + 1(.25) + 2(.05) + 3(.10) + 4B + 5C = 2.7$

$.25 + .10 + .30 + 4B + 5C = 2.7$

$.65 + 4B + 5C = 2.7$

$4B + 5C = 2.05$

$4B + 5C = 2.05$

$$\frac{\cancel{5}C}{\cancel{5}} = \frac{2.05 - 4B}{5}$$

$\rightarrow C = 0.41 - 0.8B$

$C = .41 - .8B$

$C = .41 - .8(.2)$

$C = .41 - .16$

$\boxed{C = .25}$

Sum of all Probabilities is 1.0

$P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) = 1.0$

$.15 + .25 + .05 + .10 + B + 0.41 - 0.8B = 1.0$

$0.96 + 1.0B - 0.8B = 1.0$

$0.96 + 0.2B = 1.0$

$0.2B = 0.04$

$\boxed{B = 0.2}$