MATH 1342

Section 3.1

Random Variables

A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.

A **discrete random variable** is one that can assume a countable number of possible values. A **continuous random variable** can assume any value in an interval on the number line.

A **probability distribution table of *X*** consists of all possible values of a discrete random variable with their corresponding probabilities.

Example:

Suppose a family has 3 children. Show all possible gender combinations:

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BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Example:

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

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Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

We need to know the total possible outcomes. 8 outcomes

We need to categorize them by number of girls. 0, 1, 2, 3

We need a probability of each outcome. Next slide

Example:

BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

0 girls: 1/8

1 girl: 3/8

2 girls: 3/8

3 girls: 1/8

x	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

The sum of the p-values must equal 1

(1/8) + (3/8) + (3/8) + (1/8) = 8/8 = 1

Example:

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

Find the following probabilities:

Find $P(X > 2) = P(x=3)$
$1/8$

$P(X < 1) = P(x=0)$
$1/8$

$P(1 < X \leq 3)$

0 girls:

1 girl:

$P(1 < X \leq 3) = P(x=2) + P(x=3)$

2 girls:

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

3 girls:

Example: Popper # 04

Another example: Suppose you are given the following distribution table:

X	1	2	3	4	5	6	7
$P(X)$	0.15	0.05	0.10	?	0.10	0.15	0.15

,30

Find the following:

1. $P(X = 4)$
   a. 0.10
   b. 0.15
   c. 0.30
   d. 0.05

2. $P(X < 2)$ = $P(X=1)$
   a. 0.10
   b. 0.15
   c. 0.20
   d. 0.05

3. $P(2 < X \leq 5)$
   a. 0.50
   b. 0.15
   c. 0.20
   d. 0.25

   $P(X=3) + P(X=4) + P(X=5)$
   .10 + .30 + .10

4. $P(X > 3)$ =
   a. 0.50
   b. 0.75
   c. 0.80
   d. 0.70

   $P(X=4)$ | .30
   $P(X=5)$ | .10
   $P(X=6)$ | .15
   $P(X=7)$ | .15
   .70

```
> 1-(.15+.05+.10+.10+.15+.15)
[1] 0.3
```

Expected Value:

The mean, or **expected value**, of a random variable *X* is found with the following formula:

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

Multiply each x-value by its corresponding p-value and add those products

What is the expected number of girls in the family above?

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

x	p
0	1/8
1	3/8
2	3/8
3	1/8

$$E[X] = 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8)$$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$$

What is the expected number of girls in the family above?

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

In R Studio:

```
assign("x",c(values))
assign("p",c(probabilities))
sum(x*p)
```

TI: x → L1, p → L2,
2^{nd} List (STAT), MATH (right arrow), sum (option 5)
sum(L1*L2)

```
> assign("x",c(0,1,2,3))
> assign("p",c(1/8,3/8,3/8,1/8))
> sum(x*p)
[1] 1.5
```