MATH 1342

Section 3.1

Random Variables

A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.

A **discrete random variable** is one that can assume a countable number of possible values. A **continuous random variable** can assume any value in an interval on the number line.

A **probability distribution table of *X*** consists of all possible values of a discrete random variable with their corresponding probabilities.

Example:

Suppose a family has 3 children. Show all possible gender combinations:

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BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

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Draw a probability distribution table for this example.

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Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

We need to know the total possible outcomes.

We need to categorize them by number of girls.

We need a probability of each outcome.

Example:

BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

0 girls:

1 girl:

2 girls:

3 girls:

Example:

BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Find the following probabilities:

Find $P(X > 2)$ $\quad$ $P(X < 1)$ $\quad$ $P(1 < X \leq 3)$

0 girls:

1 girl:

2 girls:

3 girls:

Another example: Suppose you are given the following distribution table:

X	1	2	3	4	5	6	7
P(*X*)	**0.15**	**0.05**	**0.10**	**?**	**0.10**	**0.15**	**0.15**

Find the following:

$P(X = 4)$ $P(X < 2)$ $P(2 < X \leq 5)$ $P(X > 3)$

Expected Value:

The mean, or **expected value**, of a random variable *X* is found with the following formula:

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

What is the expected number of girls in the family above?

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In R Studio:

```
assign("x",c(values))
assign("p",c(probabilities))
sum(x*p)
```

TI: x → L1, p → L2,
2nd List (STAT), MATH (right arrow), sum (option 5)
sum(L1*L2)

Variance and Standard Deviation

$$\sigma_X^2 = Var[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n$$

$$= \sum (x_i - \mu_X)^2 p_i$$

Or (the alternate formula)

$$\sigma_X^2 = Var[X] = E[X^2] - (E[X])^2$$

Repeat the Expectancy Formula using x^2 instead of x.

Variance and Standard Deviation

$$\sigma_X^2 = Var[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n$$

$$= \sum (x_i - \mu_X)^2 p_i$$

In R Studio: sum((x-mean)^2*p)

or

sum(x^2*p)-sum(x*p)^2

In TI: sum(L1^2*L2)-sum(L1*L2)^2

Find the **standard deviation** for the number of girls in the example above

You are at a carnival game. It costs $5 to play the game. First prize is $25 (with a probability of 0.02), second prize is $10 (with a probability of 0.05) and third prize is $5 (with a probability of 0.10). When you play the game, what are your expected winnings?

X	1	2	3	4	5	6	7
P(X)	0.15	0.05	0.10	?	0.10	0.15	0.15

What is the expected value?

The variance and standard deviation?

Rules for means and variances:

Suppose X is a random variable and we define W as a new random variable such that $W = aX + b$, where a and b are real numbers. We can find the mean and variance of W with the following formula:

$$E[W] = E[aX + b] = aE[X] + b$$

$$\sigma_W^2 = Var[W] = Var[aX + b] = a^2 Var[X]$$

Rules for means and variances:

Likewise, we have a formula for random variables that are combinations of two or more other independent random variables. Let *X* and *Y* be independent random variables,

$$E[X+Y]=E[X]+E[Y]$$

$$\sigma^2_{X+Y}=Var[X+Y]=Var[X]+Var[Y]$$

$$and$$

$$E[X-Y]=E[X-Y]=E[X]-E[Y]$$

$$\sigma^2_{X-Y}=Var[X-Y]=Var[X]+Var[Y]$$

Suppose you have a distribution, X, with mean = 22 and standard deviation = 3. Define a new random variable $Y = 3X + 1$.

a. Find the variance of X.
b. Find the mean of Y.
c. Find the variance of Y.
d. Find the standard deviation of Y.

Use the following Probability Distribution Table to find the values of A, B and C.

X	0	1	2	3	4	5
P(X)	0.15	0.25	0.05	A	B	C

Additional Information: $P(X < 4) = 0.55$; $E[X] = 2.7$