MATH 1342

Section 3.2

Bernoulli Trials

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).

2. The probability of success is p and probability of failure is $q = 1 - p$.

A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable.**

Bernoulli Experiment

A **binomial experiment** occurs when the following conditions are met:

1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials. (n)
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials. (p)

The random variable X = number of successes of a binomial experiment is a **binomial distribution** with parameters p and n where p represents the probability of a success and n is the number of trials. The possible values of X are whole numbers that range from 0 to n. As an abbreviation, we say X~B(n, p).

$0 \le X \le n$ $\quad$ $X \sim B(n, p)$

5~B(7,0.24)
This is the probability of 5 successes out of 7 trials, if the probability of success of each trial is 0.24

Binomial Distribution Formula

Binomial probabilities are calculated with the following formula:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}={}_nC_k\,p^k(1-p)^{n-k}$$

In R, $P(X=k)=$ dbinom(k,n, p). With a TI-83/84 calculator, $P(X=k)=$ binompdf(n, p, k)

These are used to calculate an exact number of successes in a binomial experiment.

Example:

$p = 0.50$ $n = 30$

A fair coin is flipped 30 times.

$x = 12$

What is the probability that the coin comes up heads exactly 12 times?

(x, n, p)

```
> dbinom(12,30,0.50)
[1] 0.08055309
```

$P(X \le k)$ = pbinom(k,n,p) $P(X \le k)$ = binomcdf(n, p, k)

What is the probability the coin comes up heads less than 12 times?

X = 0, 1, 2, 3, ..., 10, 11

largest x   n   p

```
> pbinom(11,30,0.50)
[1] 0.1002442
```

Note: "at most 12 times" means that x = 0, 1, 2, 3..., 10, 11, 12

What is the probability the coin comes up heads more than 12 times?

X = 13, 14, 15, 16, ... 29, 30

Note: "at least 12 times" means that x = 12, 13, 14, 15....30

1: X = 0, 1, 2, ... 30

throw away X = 0, 1, ... 12

```
> 1-pbinom(12,30,0.50)
[1] 0.8192027
```

Leaving X = 13, 14, 15 ... 30

Mean and Variance

The mean and variance of a binomial distribution are computed using the following formulas:

$$\mu = E\left[X\right] = np$$

$$\sigma^2 = np\left(1 - p\right)$$

```
> 30*0.50
[1] 15
> 30*.50*(1-.50)
[1] 7.5
> sqrt(7.5)
[1] 2.738613
```

$\mu =$ 15

$\sigma^2 =$ 7.5

$\sigma =$ 2.738613

The mean represents the x-value with the largest probability associated with it.
(Or the most likely to occur x-value)

Popper 05:

From text: $p=0.80$

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that: $n=5$

1. No one will contract the flu? $x=0$ B
2. All will contract the flu? $x=5$ D
3. Exactly two will get the flu? $x=2$ A
4. At least two will get the flu? $x=2,3,4,5$ C

Choices for all above questions:

a. 0.0512

b. 0.00032

c. 0.99328

d. 0.32768

```
> dbinom(0,5,0.80)
[1] 0.00032
> dbinom(5,5,0.80)
[1] 0.32768
> dbinom(2,5,0.80)
[1] 0.0512
> 1-pbinom(1,5,0.80)
[1] 0.99328
```

Example…Continued

From text:

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

Let *X* = number of family members contracting the flu. Create the probability distribution table of *X*.

```
> dbinom(1,5,0.80)
[1] 0.0064
> dbinom(3,5,0.80)
[1] 0.2048
> dbinom(4,5,0.80)
[1] 0.4096
```

X	0	1	2	3	4	5
P(X)	.00032	.0064	.0512	.2048	.4096	.32768

Find the mean and variance of this distribution.

```
> 5*0.80
[1] 4
> 5*.80*(1-.80)
[1] 0.8
> sqrt(0.80)
[1] 0.8944272
```

Mean: 4

Variance: 0.8

Standard Deviation: 0.8944