MATH 1342

Section 3.2

Bernoulli Trials

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).

2. The probability of success is p and probability of failure is $q = 1 - p$.

A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable.**

Bernoulli Experiment

A **binomial experiment** occurs when the following conditions are met:
1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).

2. There are a fixed number of trials.

3. Outcomes of different trials are independent.

4. The probability that a trial results in success is the same for all trials.

The random variable X = number of successes of a binomial experiment is a **binomial distribution** with parameters p and n where p represents the probability of a success and n is the number of trials. The possible values of X are whole numbers that range from 0 to n. As an abbreviation, we say $X{\sim}B(n, p)$.

Binomial Distribution Formula

Binomial probabilities are calculated with the following formula:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} = {}_n C_k \, p^k (1 - p)^{n-k}$$

In R, $P(X = k) = \text{dbinom}(k, n, p)$. With a TI-83/84 calculator, $P(X = k) = \text{binompdf}(n, p, k)$

Example:

A fair coin is flipped 30 times.

What is the probability that the coin comes up heads exactly 12 times?

$P(X \leq k)$ = pbinom(k,n,p) $P(X \leq k)$ = binomcdf(n, p, k)

What is the probability the coin comes up heads less than 12 times?

What is the probability the coin comes up heads more than 12 times?

Mean and Variance

The mean and variance of a binomial distribution are computed using the following formulas:

$$\mu = E[X] = np$$

$$\sigma^2 = np(1-p)$$

From text:

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

    a. No one will contract the flu?

    b. All will contract the flu?

    c. Exactly two will get the flu?

    d. At least two will get the flu?

From text:

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

Let X = number of family members contracting the flu. Create the probability distribution table of X.

Find the mean and variance of this distribution.