MATH 1342

Section 3.3

Geometric Distributions

The **geometric distribution** is the distribution produced by the random variable *X* defined to count the number of trials needed to obtain the first success.

For example:

Flipping a coin until you get a head

Rolling a die until you get a 5

A random variable *X* is geometric if the following conditions are met:

Same as binomial

1. Each observation falls into one of just two categories, "success" or "failure."
2. The probability of success is the same for each observation.
3. The observations are all independent.
4. The variable of interest is the number of trials required to obtain the first success. (No set n-value)

Notice that this is different from the binomial distribution in that the number of trials is unknown. With geometric distributions we are trying to determine how many trials are needed in order to obtain a success.

Calculating a Geometric Distribution

The probability that the first success occurs on the n^{th} trial is

$$P(X = n) = (1 - p)^{n-1} p$$

where p is the probability of success.

The probability that it takes *more* than n trials to see the first success is

$$P(X > n) = (1 - p)^n$$

exact x-value

more than

R commands: $P(X = n) =$ dgeom($n - 1$, p) and $P(X > n) = $ 1-pgeom($n - 1$, p)

Ti-83/84 commands: $P(X = n) =$ geometpdf(p, n) and $P(X > n) =$ 1- geometcdf (p, n)

less than: without the "1-"

Mean and Variance

The mean, or expected number of trials to get a success in a geometric distribution is $E[X] = \mu = \dfrac{1}{p}$ and the variance is $\sigma^2 = \dfrac{1-p}{p^2}$.

"at most 5": x = 0,1,2,3,4,5
"as least 5": x = 5, 6, 7, 8, 9…
"less than 5": x = 0,1,2,3,4
"more than 5": x = 6, 7, 8, 9…

Examples:

From text: #**8.** $p = 0.44$

A quarter back completes 44% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass.

a. What is the probability that the quarterback throws 3 incomplete passes before he has a completion? First success is on 4^{th} try

b. How many passes can the quarterback expect to throw before he completes a pass? $\mu = E[X] = \frac{1}{p}$

c. Determine the probability that it takes more than 5 attempts before he completes a pass. $X = 6, 7, 8, \ldots$

d. What is the probability that he attempts more than 7 passes before he completes one? $X = 8, 9, 10, \ldots$

a)

```
> dgeom(4-1,0.44)
[1] 0.07727104
```

b) $\mu = \frac{1}{p} = \frac{1}{.44}$

```
> 1/.44
[1] 2.272727
```

(In reality, this means that most likely the second pass attempt will be the first completed pass)

c)

```
> 1-pgeom(5-1,0.44)
[1] 0.05507318
```

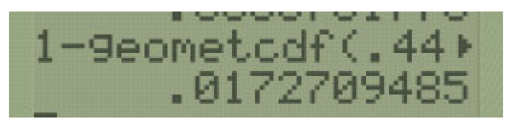
$x = 1, 2, 3 \ldots$

throw away $x = 1, 2, 3, 4, 5$

leaving: $x = 6, 7, 8, \ldots$

d)

```
1-pgeom(7-1,0.44)
1] 0.01727095
```



Popper 06:

Newsweek in 1989 reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming a random sample from the population of all school children at risk,

find: (Looking for 5 out of 10)

1. The probability that at least 5 children out of 10 in a sample taken from a school may have a blood lead level that may impair development.

Binomial: X = 5, 6, 7, ... 10

```
1-pbinom(4,10,0.60)
1] 0.8337614
```

D

2. The probability you will need to test 10 children before finding a child with a blood lead level that may impair development (10^{th} child his the first with impairment).

Geometric: X = 10

```
dgeom(10-1,0.60)
1] 0.0001572864
```

B

3. The probability you will need to test no more than 10 children before finding a child with a blood lead level that may impair development.

Geometric X = 1, 2, 3 ... 10

```
pgeom(10-1,0.60)
1] 0.9998951
```

A

a. 0.9998951 b. 0.0001572864

c. 0.98976 d. 0.8337614