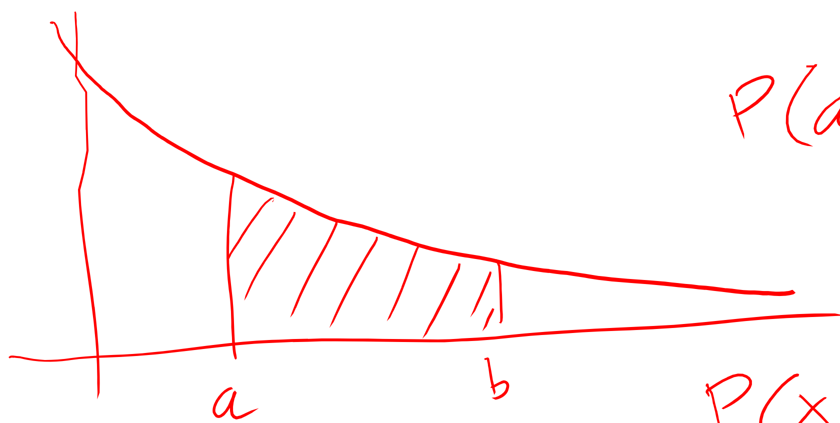
MATH 1342

Section 4.1

Density Curves

A *density curve* is a graph whose area between it and the x-axis is equal to one. These graphs come is a variety of shapes but the most familiar “normal” graph is bell shaped. The area under the curve in a range of values indicates the proportion of values in that range.

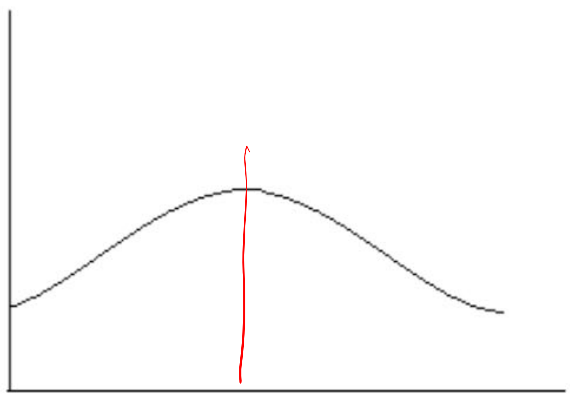


$P(a<x<b) =$ shaded area

$P(x>a) = P(x \geq a)$

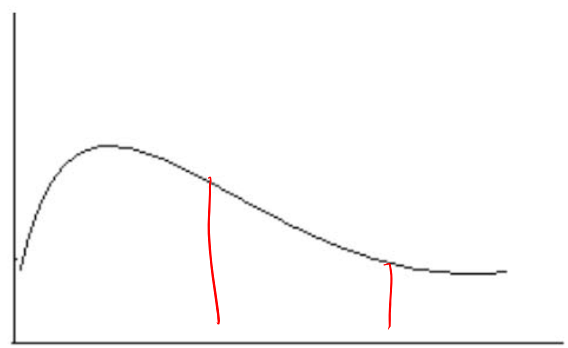
Skewness and curves:

Bell Shaped (normal)



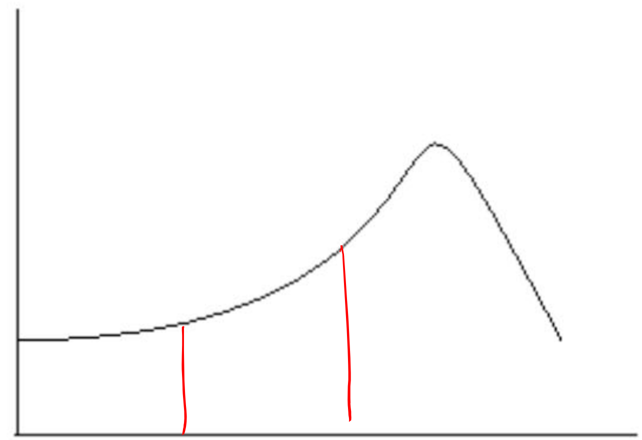
Mean = Median

Skewed Right



Median < Mean

Skewed Left

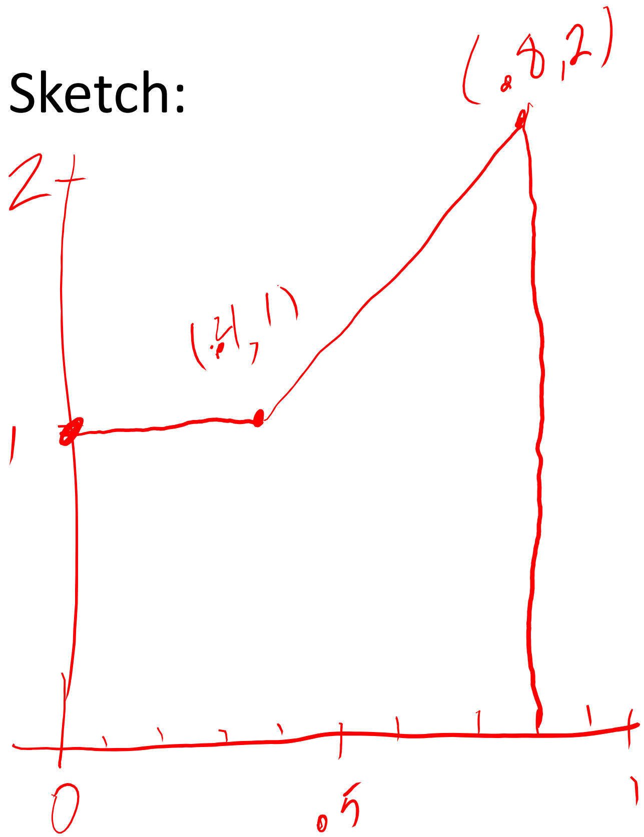


Mean < Median

Example: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.

$\overset{x}{\leftrightarrow}$ $\updownarrow y$

Sketch:



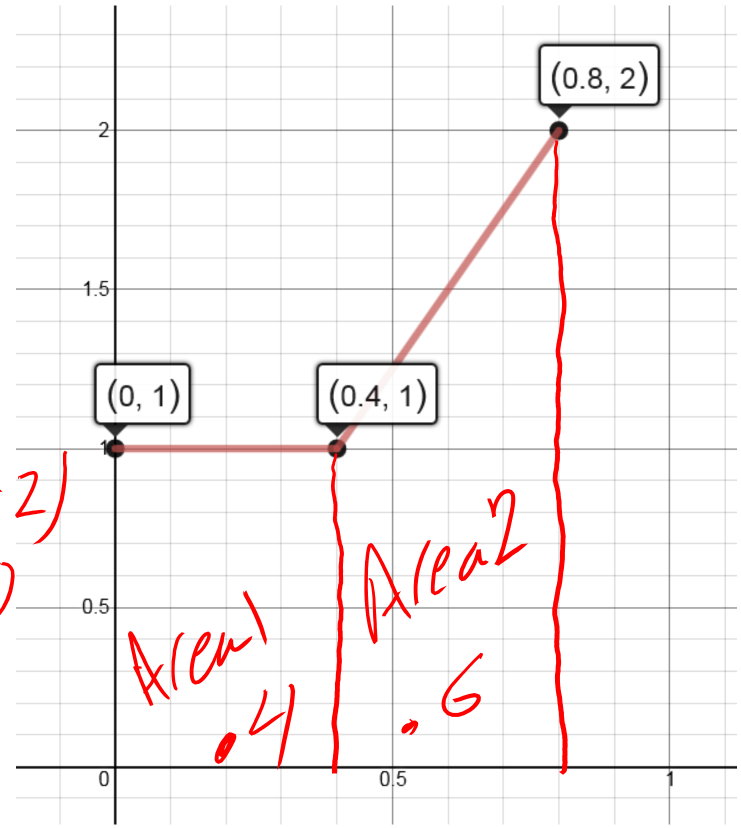
Area Formulas:
Rectangle: Area = base * height
Triangle: Area = ½ * base * height
Trapezoid: Area = ½ * height * (base1 + base2)
[bases are parallel sides, height is distance between them]

Measure: top – bottom (y-values); right – left (x-values)

Example: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.

Total Area should equal 1

Sketch:



Area1: $A = b \cdot h = (.4)(1) = .4$

Base: $0.4 - 0 = .4$

Height: $1 - 0 = 1$

Area 2: $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(.4)(1+2)$

Base1: $1 - 0 = 1$ $\quad \frac{1}{2}(.4)(3)$

Base2: $2 - 0 = 2$ $\quad = .6$

height: $.8 - .4 = .4$

Total Area: $.4 + .6 = 1$

Example: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.

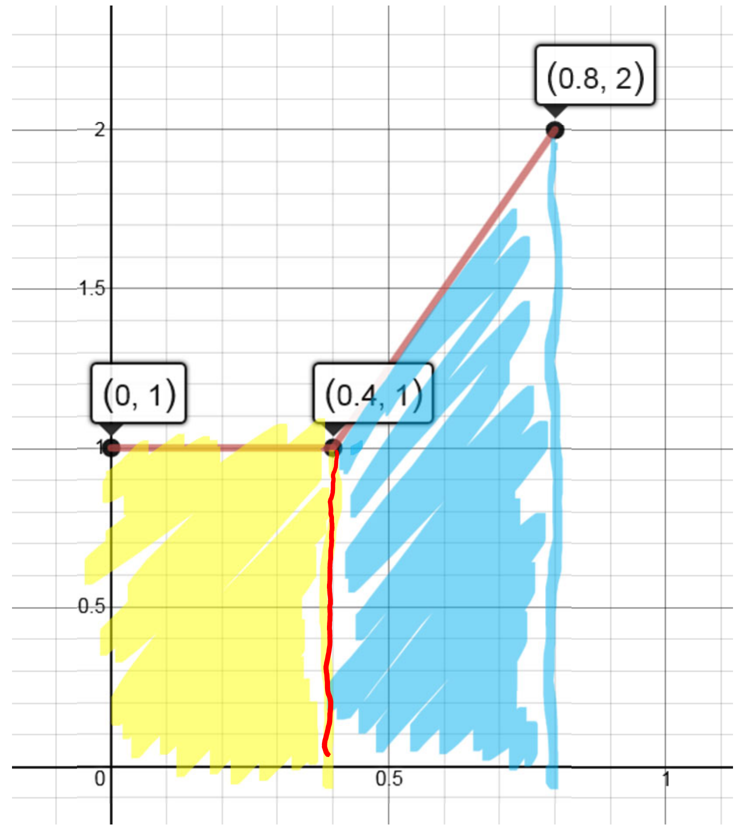
What percent of observations fall below .4? $P(X \leq .4) = .4$

40%

What percent of observations lie between .4 and .8? $P(.4 \leq X \leq .8) = .6$

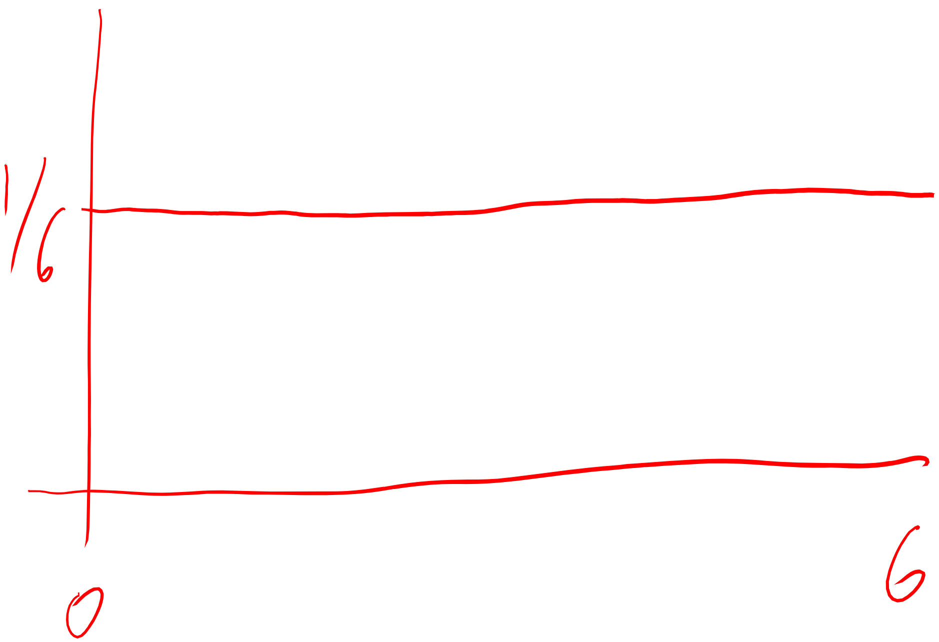
60%

What percent of observations are equal to .4? $P(X = .4) = 0$



Example: Consider a uniform density curve defined from x = 0 to x = 6.

Sketch:



Base: $6 - 0 = 6$

Area: 1

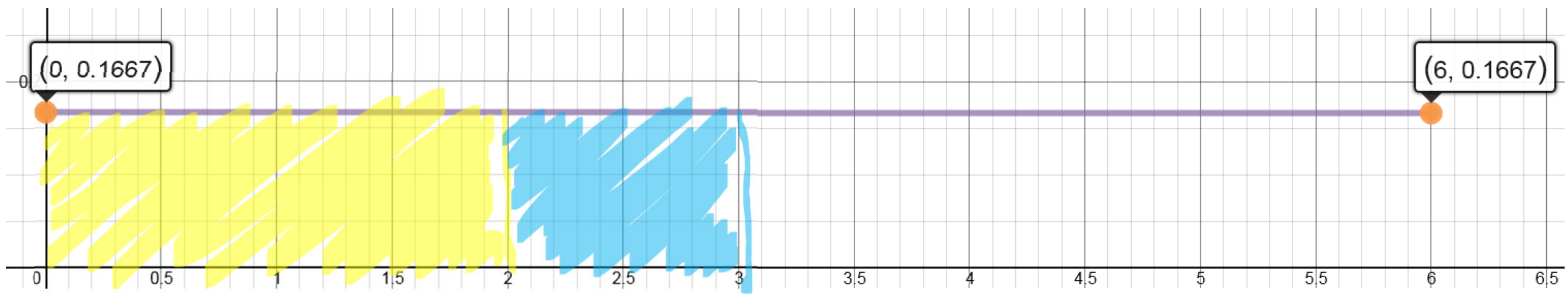
$A = b \cdot h$

$1 = 6 \cdot h$

$\frac{1}{6} = h$

Example: Consider a uniform density curve defined from $x = 0$ to $x = 6$.

Sketch:



① $P(X<2) = (2)(1/6) = .333$

Base: $2-0=2$

height: $1/6$

② $P(2<X<3) = 1(1/6) = .1667$

Base: $3-2=1$

height: $1/6$

③ #1 + #2 = $1/3 + 1/6 = 1/2 = .5$ Median: 3

Popper 08:

1. What percent of observations fall below 2?

   a. 0.167 b. 0.333 c. 0.5 d. 2.0

2. What percent of observations lie between 2 and 3?

   a. 0.167 b. 0.333 c. 0.5 d. 2.0

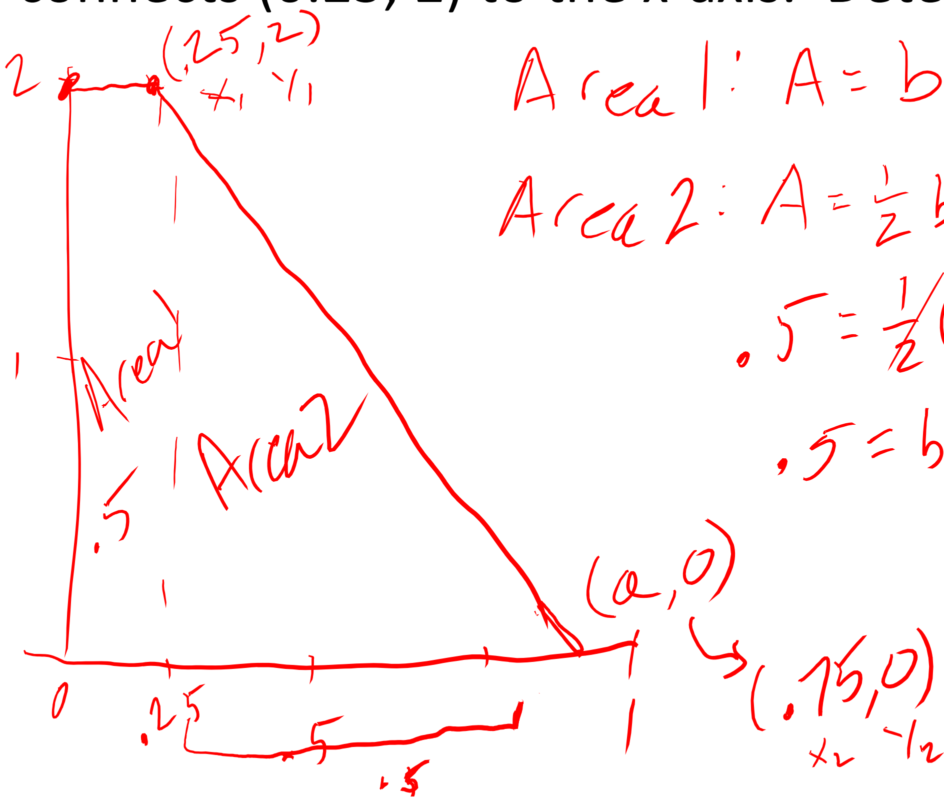
3. Find the median.

   $\frac{6+0}{2} = 3$ (uniform)

   a. 1/6 b. 2 c. 3 d. No Median

Another Example:

A probability density curve consists of two line segments. One segment connects the points (0, 2) and (0.25, 2), and the other connects (0.25, 2) to the x-axis. Determine P(x > 0.35).



Area 1: $A = b \cdot h = (.25)(2) = .5$

$A_1 + A_2 = 1$

$.5 + A_2 = 1$

$\downarrow$

$.5$

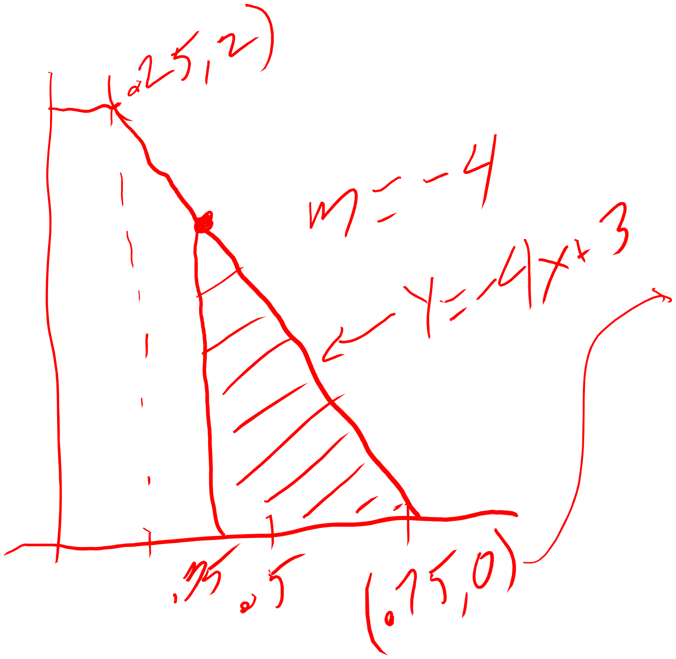
Area 2: $A = \frac{1}{2} b \cdot h$

$.5 = \frac{1}{\cancel{2}}(b)(\cancel{2})$

$.5 = b$

slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{.75-.25}$

$m = \frac{-2}{.5} = -4$



$y = mx + b$

$y = -4x + b$

$0 = -4(.75) + b$

$3 = b$

$P(X > .35) = \frac{1}{2}(.4)(1.6) = \boxed{0.32}$

Base: $.75 - .35 = .4$

height: $y = -4(.35) + 3 = 1.6$

FYI:

Median: $x = .25$

(.5 Area left

.5 Area right)