MATH 1342

Section 7.4

Confidence Interval for a Population Mean

Recall that formula for a confidence interval is *statistic* $\pm$ *margin of error*. When we are making an inference about a population mean, the statistic will be our sample mean, $\bar{x}$.
The critical value we use to find the margin of error for our calculation will be based on whether the population or sample standard deviation is known. When the population standard deviation is known,

we use the formula $\bar{x} \pm z * \dfrac{\sigma}{\sqrt{n}}$ and when it is unknown, we will need to find the sample standard

deviation, *s*, and use the formula $\bar{x} \pm t * \dfrac{s}{\sqrt{n}}$ where *t** is the *t*-critical value based on *n* – 1 degrees of

freedom.

Confidence Intervals

General formula: *statistic ± margin of error*

One-sample z-test: $\bar{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$

Two-proportion z-test: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

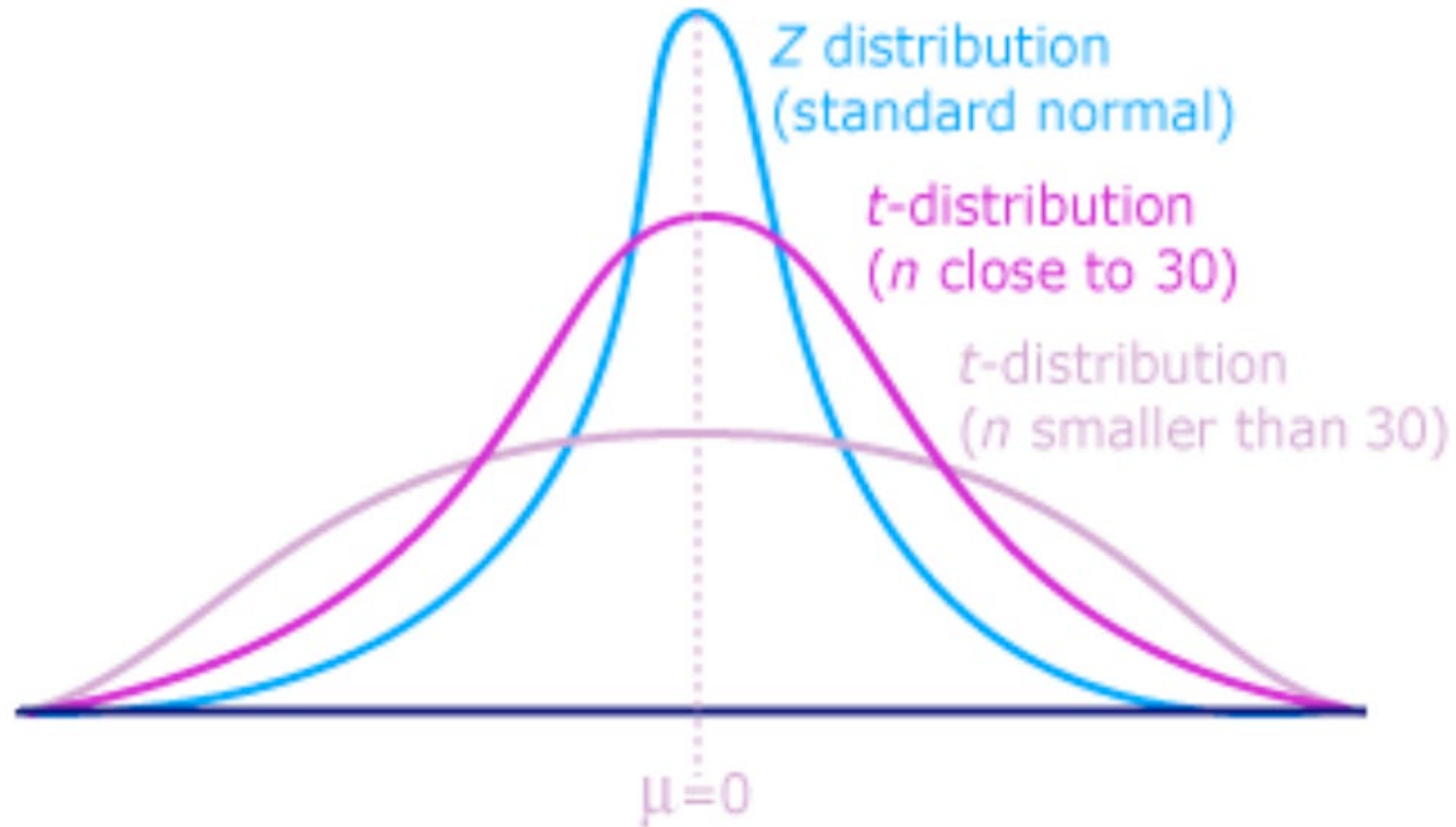One-sample t-test: $\bar{x} \pm t^* \dfrac{s}{\sqrt{n}}$

One-proportion z-test: $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Two-sample z-test: $(\bar{x}_1 - \bar{x}_2) \pm z^* \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Two-sample t-test: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

So, what is *t**?

t-distribution vs. standard normal distribution:

How do we find critical values for a *t*-distribution?

Look at the online text book, under appendices.

Degrees of freedom are on the left and the top end (1-confidence level/2) are given at the top.

df	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
						Upper tail probability p						
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	4.8487	6.9646	9.9248	14.0890	22.3271	31.5991
3	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	3.4819	4.5407	5.8409	7.4533	10.2145	12.9240
4	0.7407	0.9410	1.1896	1.5332	2.1318	2.7764	2.9985	3.7469	4.6041	5.5976	7.1732	8.6103
5	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	2.7565	3.3649	4.0321	4.7733	5.8934	6.8688
6	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	2.6122	3.1427	3.7074	4.3168	5.2076	5.9588
7	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.5168	2.9980	3.4995	4.0293	4.7853	5.4079
8	0.7064	0.8889	1.1081	1.3968	1.8595	2.3060	2.4490	2.8965	3.3554	3.8325	4.5008	5.0413
9	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.3984	2.8214	3.2498	3.6897	4.2968	4.7809
10	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.3593	2.7638	3.1693	3.5814	4.1437	4.5869
11	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.3281	2.7181	3.1058	3.4966	4.0247	4.4370
12	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.3027	2.6810	3.0545	3.4284	3.9296	4.3178
13	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.2816	2.6503	3.0123	3.3725	3.8520	4.2208
14	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.2638	2.6245	2.9768	3.3257	3.7874	4.1405
15	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.2485	2.6025	2.9467	3.2860	3.7328	4.0728
16	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208	3.2520	3.6862	4.0150

Assumptions:

The assumptions for a population mean are:
1. The sample must be an SRS from the population of interest.
2. The data must come from a normally distributed population. If this is not the case or if we are unsure whether the population is normally distributed, the sampling distribution of $\bar{x}$ must be normally distributed. (Recall from section 4.4 that we can assume that the sampling distribution of $\bar{x}$ is normal for values of n greater than 30.)

Changes is the Confidence Interval:

The following will cause an increase in the width of the confidence interval:

- An increase in the Margin of Error

- An increase in the Standard Deviation (σ or s).

- An increase in the Confidence Level.

- A decrease in the Sample Size.

Note: Changes in the sample mean ($\bar{x}$) will affect the center of the interval but will have *no effect* on the interval width.

Examples:

1. Suppose your class is investigating the weights of Snickers 1-ounce fun-size candy bars to see if customers are getting full value for their money. Assume that the weights are normally distributed with standard deviation $\sigma = .005$ ounces. Several candy bars are randomly selected and weighed with sensitive balances borrowed from the physics lab.

The weights are: .95   1.02   .98   .97   1.05   1.01   .98   1.00

We want to determine a 90% confidence interval for the true mean, μ.

a. What is the sample mean?

b. Determine z^*.

c. Determine the 90% confidence interval. (Show your work)

d. Write a sentence that explains the significance of the confidence interval.

You select a sample of 50 people with a mean height of 72 inches from a population that has a standard deviation of 3 inches.

What would the margin of error be for a 95% confidence interval?

What is the width of confidence interval?

What is the confidence interval?

Give an interpretation of this interval.

What will cause the width of the confidence interval to increase?

Examples:

A SRS of 16 seniors from HISD had a mean SAT-math score of 500 and a standard deviation of 100. We know that the population of SAT-math scores for seniors in the district is approximately normally distributed.

Find the 90% confidence interval for the mean SAT-math score for the population of all seniors in the district.

t-score table:

Table entry for p and C is the critical value t* with probability p lying to its right and probability C lying between –t* and t*

Probability p

t*
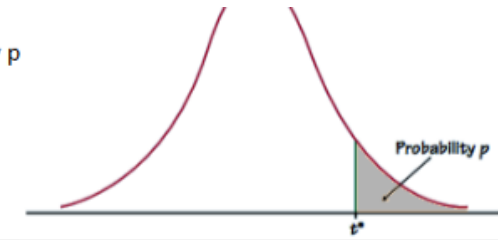
df	Upper tail probability p											
	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
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16	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208	3.2520	3.6862	4.0150
17	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.2238	2.5669	2.8982	3.2224	3.6458	3.9651
18	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.2137	2.5524	2.8784	3.1966	3.6105	3.9216
19	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.2047	2.5395	2.8609	3.1737	3.5794	3.8834
20	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.1967	2.5280	2.8453	3.1534	3.5518	3.8495
21	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.1894	2.5176	2.8314	3.1352	3.5272	3.8193
22	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.1829	2.5083	2.8188	3.1188	3.5050	3.7921
23	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.1770	2.4999	2.8073	3.1040	3.4850	3.7676
24	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.1715	2.4922	2.7969	3.0905	3.4668	3.7454
25	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.1666	2.4851	2.7874	3.0782	3.4502	3.7251
26	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.1620	2.4786	2.7787	3.0669	3.4350	3.7066
27	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.1578	2.4727	2.7707	3.0565	3.4210	3.6896
28	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.1539	2.4671	2.7633	3.0469	3.4082	3.6739
29	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.1503	2.4620	2.7564	3.0380	3.3962	3.6594
30	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.1470	2.4573	2.7500	3.0298	3.3852	3.6460
40	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.1229	2.4233	2.7045	2.9712	3.3069	3.5510
50	0.6794	0.8489	1.0473	1.2987	1.6759	2.0086	2.1087	2.4033	2.6778	2.9370	3.2614	3.4960
60	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.0994	2.3901	2.6603	2.9146	3.2317	3.4602
80	0.6776	0.8461	1.0432	1.2922	1.6641	1.9901	2.0878	2.3739	2.6387	2.8870	3.1953	3.4163
100	0.6770	0.8452	1.0418	1.2901	1.6602	1.9840	2.0809	2.3642	2.6259	2.8707	3.1737	3.3905
1000	0.6747	0.8420	1.0370	1.2824	1.6464	1.9623	2.0564	2.3301	2.5808	2.8133	3.0984	3.3003
z*	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.0537	2.3263	2.5758	2.8070	3.0902	3.2905
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence Level C

t-score table:
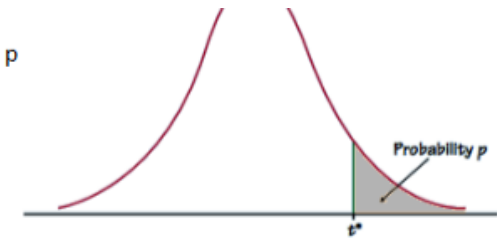
Since n = 16, we are going to use the value for n − 1 = 15 degrees of freedom.

Since we want a confidence level of 90%, we will use an "Upper Tail Probability" of (1 − 0.90)/2 = 0.05.

This is to account for only using one tail end of our graph, rather than 2.

Our t* value will be 1.7531.

Table entry for p and C is the critical value t* with probability p lying to its right and probability C lying between −t* and t*

Probability p

t*

						Upper tail probability p						
df	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
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9	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.3984	2.8214	3.2498	3.6897	4.2968	4.7809
10	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.3593	2.7638	3.1693	3.5814	4.1437	4.5869
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12	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.3027	2.6810	3.0545	3.4284	3.9296	4.3178
13	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.2816	2.6503	3.0123	3.3725	3.8520	4.2208
14	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.2638	2.6245	2.9768	3.3257	3.7874	4.1405
15	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.2485	2.6025	2.9467	3.2860	3.7328	4.0728
16	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208	3.2520	3.6862	4.0150
17	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.2238	2.5669	2.8982	3.2224	3.6458	3.9651
18	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.2137	2.5524	2.8784	3.1966	3.6105	3.9216
19	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.2047	2.5395	2.8609	3.1737	3.5794	3.8834
20	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.1967	2.5280	2.8453	3.1534	3.5518	3.8495
21	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.1894	2.5176	2.8314	3.1352	3.5272	3.8193
22	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.1829	2.5083	2.8188	3.1188	3.5050	3.7921
23	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.1770	2.4999	2.8073	3.1040	3.4850	3.7676
24	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.1715	2.4922	2.7969	3.0905	3.4668	3.7454
25	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.1666	2.4851	2.7874	3.0782	3.4502	3.7251
26	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.1620	2.4786	2.7787	3.0669	3.4350	3.7066
27	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.1578	2.4727	2.7707	3.0565	3.4210	3.6896
28	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.1539	2.4671	2.7633	3.0469	3.4082	3.6739
29	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.1503	2.4620	2.7564	3.0380	3.3962	3.6594
30	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.1470	2.4573	2.7500	3.0298	3.3852	3.6460
40	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.1229	2.4233	2.7045	2.9712	3.3069	3.5510
50	0.6794	0.8489	1.0473	1.2987	1.6759	2.0086	2.1087	2.4033	2.6778	2.9370	3.2614	3.4960
60	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.0994	2.3901	2.6603	2.9146	3.2317	3.4602
80	0.6776	0.8461	1.0432	1.2922	1.6641	1.9901	2.0878	2.3739	2.6387	2.8870	3.1953	3.4163
100	0.6770	0.8452	1.0418	1.2901	1.6602	1.9840	2.0809	2.3642	2.6259	2.8707	3.1737	3.3905
1000	0.6747	0.8420	1.0370	1.2824	1.6464	1.9623	2.0564	2.3301	2.5808	2.8133	3.0984	3.3003
z*	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.0537	2.3263	2.5758	2.8070	3.0902	3.2905
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence Level C

Using Rstudio and TI to calculate t*

In Rstudio:

qt(1.##/2,df)

so for a 90% confidence interval with a sample size of 16: qt(1.90/2,15)

In TI 84 (this command does not appear in TI 83)

Under the DIST menu, select InvT

area: 1.90/2

df: 15

Confidence Interval Calculation:

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

Look at the following example:

The effect of exercise on the amount of lactic acid in the blood was examined in an article for an exercise and sport magazine. Eight males were selected at random from those attending a week-long training camp. Blood lactate levels were measured before and after playing three games of racquetball, as shown in the accompanying table. Use this data to estimate the mean increase in blood lactate level using a 95% confidence interval.

Player	1	2	3	4	5	6	7	8
Before	13	20	17	13	13	16	15	16
After	18	37	40	35	30	20	33	19

Interpretation of the Confidence Interval:

Example:

A 95% confidence interval for the mean of a population is to be constructed and must be accurate to within 0.3 unit. A preliminary sample standard deviation is 2.9. Find the smallest sample size n that provides the desired accuracy.

When to use z* or t*?

Use z* if:

the question involved proportions

you are given standard deviation of the population (σ).

you are asked to find sample size.

Use t* if:

you are not provided with standard deviation of population.

   (you may be given standard deviation of your sample

   or given data to calculate standard deviation.)

Is it a z* or a t*?

- You wish to determine the confidence interval for the mean water salinity in a certain town. Preliminary studies have shown that water salinity is normally distributed with a standard deviation of 0.03. You select a sample of 25 households, and determine that the mean water salinity is 18 parts per thousand, with a standard deviation of 0.04. Construct the 90% confidence interval for the mean of water salinity.

Is it z* or t*?

- In a sample of 50 examinees, you have determined the mean score to be 87 with a standard deviation of 2. Determine the 95% confidence interval for the mean score of the population.

Is it t* or z*?

- You are designing a study to determine the interaction between sugar intake and insomnia. To do this, you wish to construct a sample of subjects that will have large amounts of sugar prior to bed and measure the duration of their sleep. Preliminary studies show that this will have a standard deviation of 0.5 hours. What size interval must be used to have a margin of error of no more than 1 hour?

A group of 10 students are comparing exam scores for a test they recently took. Based on these results, you want to find out, with 90% confidence, what range of grades you can expect to receive.

Exam Scores: 70, 75, 80, 87, 90, 90, 93, 94, 96, 100

What is the sample mean?

What is the sample standard deviation?

What is the t* value that can be used here?

What is the margin of error?

What is the confidence interval?