MATH 3307

Final Exam Review

Final Exam will be 24 Questions (Equally Weighted)
worth 80 points.
Essay (distributed prior) to be handed in that day.

The probability that a randomly selected person is a vegetarian is P(V) = 0.07 and the probability that a randomly selected person exercises regularly is P(E) = 0.35. The probability that a randomly selected person is a vegetarian and exercises regularly is 0.04. Find the probability that a randomly selected person either is a vegetarian, a regular exerciser, or both.

$P(V \cup E) = P(V) + P(E) - P(V \cap E) = 0.07 + 0.35 - 0.04 = 0.38$

From the Previous Example:

Determine the probability that a vegetarian is also someone who exercises.

$$P(E|V) = \frac{P(E \cap V)}{P(V)} = \frac{0.04}{0.07} = 0.0571$$

Are being a vegetarian and exercising independent events?

$$P(E) \neq P(E|V)$$

0.35 ≠ 0.571

Not Independent

You are dealt 7 cards from a standard deck of 52 cards. What is the probability that your hand will contain exactly 2 aces or 2 kings or both?

$$P(2A) + P(2K) - P(2A \text{ and } 2K)$$

$$\frac{{}_{4}C_{2} \cdot {}_{48}C_{5} + {}_{4}C_{2} \cdot {}_{48}C_{5} + {}_{4}C_{2} \cdot {}_{4}C_{2} \cdot {}_{44}C_{3}}{{}_{52}C_{7}} = 0.150$$

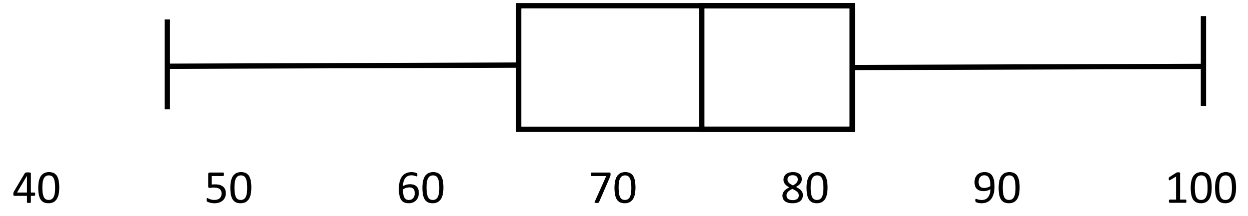
After analyzing a set of data, the Five Number Summary was determined to be:

47 67 74 81 100

- Determine the range. Max – Min = 100 – 47 = 53
- Determine the interquartile range. Q3 – Q1 = 81 – 67 = 14
- Are there any outliers? If so, where? No Outliers
- Sketch a box and whisker plot of the data.

Outliers: Lower: Q1 – 1.5(IQR) = 67 – 1.5(14) = 46

Upper: Q3 + 1.5(IQR) = 81 + 1.5(14) = 102



Data collected revealed a distribution that was left skewed with a lower quartile of 43 and an upper quartile of 56.

- On what range values must the median fall?

  Between 43 and 56

- On what range of values must the mean fall?

  Less than the median (since the median is not know, less than 56)

A maintenance manager is interested in which entrance is most often used in a building. To determine this, he counts how many people use each entrance during the course of one day.

- Is this an observational study or an experiment?

This is an observational study. There is no control on the part of the researcher.

A researcher is interested in whether the choice of exit from a building is different in an emergency. To do this, two groups are brought into a building to listen to a presentation. One group listens to the entire presentation and then leaves, with their choice of exit being recorded. The other group's presentation is interrupted by the fire alarm, and their choice of exit is recorded. The results are then compared.

- Is this study an experiment or an observational study?

  Experiment. The researcher is exercising control over the situation

- Which of these two groups would be considered the control group?

  Control Group: The groups that did not have the fire alarm go off. (No treatment)

In selection a sample from a population, a researcher does takes the following actions. Identify the sampling technique used for each.

- Selections are made from a random digit table with all members of population receiving equal representation. **Simple Random Sample**
- Selections are made from a random digit table where people over the age of 60 are twice as likely to be selected than anyone else. **Probability Sample**
- The sample is formed from people that filled out a survey from a website that was advertised at the supermarket. **Voluntary Sample**
- To make the population size more manageable, only people with birthdays in February were considered. A random selection was made from them. **Multi-Phase Sample**
- Exactly half the sample was randomly selected from the population members under the age of 25, and the other half was randomly selected from those over 25. **Stratified Sample**

Based on the following probability distribution, what will be the mean and the standard deviation?

X	50	25	15	10	5
P(X)	0.01	0.05	0.1	0.2	P(A)

```
> assign("p",c(.01,.05,.1,.2))
> 1-sum(p)
[1] 0.64
> assign("p",c(.01,.05,.1,.2,.64))
> sum(p)
[1] 1
> assign("x",c(50,25,15,10,5))
> sum(x*p)
[1] 8.45
> sum(x^2*p)-sum(x*p)^2
[1] 43.3475
> sqrt(43.3475)
[1] 6.583882
```

Mean: 8.45
Standard Deviation: 6.584

The probability of a new car battery failing is 3%. You sold 25 car batteries in a given week. What is the probability that <u>exactly 2</u> of them would have failed.

$x = 2$
$n = 25$
$p = 0.03$

```
> dbinom(2,25,0.03)
[1] 0.1340027
```

The probability of someone being allergic to peanuts is 8%. In a room of 75 people, what is the probability that less than 3 have a peanut allergy?

```
> pbinom(2,75,.08)
[1] 0.05482819
```

The probability of winning one round of a card game in 0.24. What is the probability that your first win will be on the second round of the game?

```
dgeom(2-1,0.24)
1] 0.1824
```

The amount of money spent on lunch is normally distributed with a mean of $7.50 and a standard deviation of $1.10. What is the probability that someone will spend more than $12.00 on lunch?

$\mu = 7.50$
$\sigma = 1.10$
$x > 12$

```
> 1-pnorm(12,7.50,1.10)
[1] 2.148428e-05
```

0.0000215

Find a value of c so that $P(Z > c) =$ **0.785.**

```
> qnorm(1-0.785)
[1] -0.7891917
```

The mean height of students in a fifth grade class is 52 inches with a standard deviation of 1.6 inches. You select 5 students from this class. What is the probability that the mean height of your selection is between 48 and 50 inches?

$x = 48$ to 50

$\mu = 52$

$\sigma = {}^{1.6}/_{\sqrt{5}}$

```
> pnorm(50,52,1.6/sqrt(5))-pnorm(48,52,1.6/sqrt(5))
[1] 0.002594292
```

In a large population, 32% of the adults own pets. A simple random sample of 200 adults is to be contacted and the sample proportion computed. What is the mean and standard deviation of the sampling distribution of the sample proportions?

$$\mu = p = 0.32$$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.32(1-0.32)}{200}}$$

```
> sqrt(.32*(1-.32)/200)
[1] 0.03298485
```

In a large population, 32% of the adults own pets. A simple random sample of 200 adults is to be contacted and the sample proportion computed. Find the probability that **<u>less than</u>** 50 of this sample own pets?

x = 50/200
μ = 0.32
σ = 0.03298 (previous slide)

```
> pnorm(50/200,0.32,0.03298485)
[1] 0.01691104
```

Determine the correlation coefficient of the following data:

x	3	6	8	10	11	14	19
y	238	204	177	150	122	103	85

```
> assign("x",c(3,6,8,10,11,14,19))
> assign("y",c(238,204,177,150,122,103,85))
> cor(x,y)
[1] -0.9628893
```

Determine the LSRL of the data:

x	3	6	8	10	11	14	19
y	238	204	177	150	122	103	85

```
> lm(y~x)

Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)            x
     256.94       -10.14
```

$$\hat{y} = -10.14x + 256.94$$

or

$$\hat{y} = 256.94 - 10.14x$$

Determine the residual of the data for the value of x = 14.

x	3	6	8	10	11	14	19
y	238	204	177	150	122	103	85

$y - \hat{y}$

```
> 103-(-10.14*14+256.94)
[1] -11.98
> residuals(lm(y~x))
         1          2          3          4          5          6          7
 11.462329   7.868151   1.138699  -5.590753 -23.455479 -12.049658  20.626712
```

A simple random sample of 75 children indicated that 12% come from single-parent homes. Create the 95% confidence level of the true population proportion for children from single-parent homes.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

```
> .12-qnorm(1.95/2)*sqrt(.12*(1-.12)/75)
[1] 0.04645563
> .12+qnorm(1.95/2)*sqrt(.12*(1-.12)/75)
[1] 0.1935444
```

[0.046, 0.194]

The height of 8^{th} graders is known to have a standard deviation of 4 inches. [**A simple random sample of 81 of them is chosen and found to have a mean height of 52 inches.**] Construct a 89% confidence interval for the mean height of 8^{th} grade students.

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

```
> 52-qnorm(1.89/2)*4/sqrt(81)
[1] 51.28969
> 52+qnorm(1.89/2)*4/sqrt(81)
[1] 52.71031
```

[51.29, 52.71]

In order to construct a confidence interval with a level of 98% for a population proportion, how what is the minimum sample size you would need in order for the margin of error to be less than 1.5%?

$\hat{p} = 0.50$

```
.5*(1-.5)*(qnorm(1.98/2)/.015)^2
] 6013.216
```

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\frac{ME}{z^*} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\left(\frac{ME}{z^*}\right)^2 = \frac{\hat{p}(1-\hat{p})}{n}$$

$$\left(\frac{z^*}{ME}\right)^2 = \frac{n}{\hat{p}(1-\hat{p})}$$

$$\hat{p}(1-\hat{p})\left(\frac{z^*}{ME}\right)^2 = n$$

n = 6014

A school district claims that their male and female students have equal grades on final exams. Out of 55 males surveyed the mean final exam grade was 78 with a standard deviation of 3.5. Out of 62 females surveyed, the mean score was 82 with a standard deviation of 6.1. Test the school district's claim at a confidence level of 5%.

Ho: $\mu_1 = \mu_2$

Ha: $\mu_1 \neq \mu_2$

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

```
> (78-82)/sqrt(3.5^2/55+6.1^2/62)
[1] -4.409501
> 2*pt(-4.409501,54)
[1] 4.974561e-05
```

p = 0.00004975

$p < \alpha$ RHo