

MATH 3307

Final Exam Review

Final Exam will be 24 Questions (Equally Weighted)
worth 80 points.

Essay (distributed prior) to be handed in that day.

The probability that a randomly selected person is left handed (the event L) is $P(L) = 0.1$ and the probability that a randomly selected person is a blonde (the event B) is $P(B) = 0.3$. The probability that a randomly selected person is left handed and a blonde is 0.02. Find the probability that a randomly selected person either is left handed, a blonde, or both. $P(L \cap B) = 0.02$

$$P(L \cup B) = P(L) + P(B) - P(L \cap B)$$
$$= 0.1 + 0.3 - 0.02 = 0.38$$

$$P(L \cup B) = 0.38$$

Continued from Previous:

Find the probability that someone is left-handed, given that they are blonde?

$$P(L|B) = \frac{P(L \cap B)}{P(B)} = \frac{.02}{.3} = 0.067$$

Are left-handedness and blonde hair independent? No

$$P(L) \neq P(L|B) \\ .1 \neq .067$$

You are dealt a hand of seven cards from a standard deck. What is probability that your hand will contain all cards of the same suit? (Hint: There are four suits of 13 cards each, for a total of 52 cards.)

$$\text{Total: } 52C_7$$

$$P(\text{All one suit}) = \frac{4 \cdot 13C_7}{52C_7}$$

$$\begin{aligned} \text{selection: } & 13C_7 + 13C_0 + 13C_0 + 13C_0 \\ & + 13C_0 + 13C_7 + 13C_0 + 13C_0 \\ & + 13C_0 + 13C_0 + 13C_7 + 13C_0 \\ & + 13C_0 + 13C_0 + 13C_0 + 13C_7 \end{aligned}$$

```
> (4*choose(13,7))/choose(52,7)
[1] 5.130637e-05
```

0.0000513

$$4 \cdot 13C_7$$

Sample data was collected and analyzed to show the following:

Mean: 75.4

Median: 85.3

Lower Quartile: 72.8

Upper Quartile: 91.3

Minimum: 45.0

Maximum: 118.0

Determine the following:

Range $Max - Min = 118 - 45 = 73$

Interquartile Range $Q_3 - Q_1 = 91.3 - 72.8$

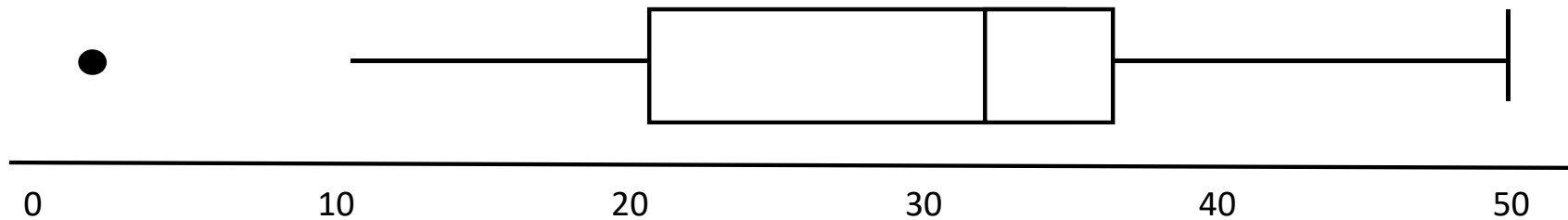
Are there outliers? (If yes, where?) $= 18.5$

Skewedness (Left, Right, Symmetric)

Outliers: Low side: $Q_1 - 1.5(IQR) = 72.8 - 1.5(18.5) = 45.05$

Upper side: $Q_3 + 1.5(IQR) = 91.3 + 1.5(18.5) = 119.05$

Based on the following Box and Whisker plot, determine the following (if possible):



Mean : *Cannot be determined*

Median = *32*

Lower Quartile = *21*

Upper Quartile = *37*

Range = *50 - 2 = 48*

Interquartile Range = *37 - 21 = 16*

You wish to select a sample of 60 individuals [40 men and 20 women] from a population of 500 men and 500 women. Identify the following sampling techniques used below:

- All 1000 names were placed in a hat and 60 were selected for the sample.

Simple Random Sample

- The names of the 500 men were placed in a hat and 40 were selected. Next, the names of 500 women were placed in a hat and 20 were selected.

Stratified Sample

- All 500 women had their names placed in a hat once and all 500 men had their names placed in the hat twice. From here, 60 were selected.

Probability Sample

- From an alphabetized list of the 1000 people, the first 60 names were used.

Convenience Sample

In a study to determine if classical music or contemporary music will help study retention, 120 students were selected and divided into three groups. One group listened to classical music while studying and another listened to contemporary music. The third group was a control group. Afterwards, their exam scores based on the material they studied was compared.

- **Is this study an experiment or an observational study? (How can this study be revised to represent the opposite choice?)**

Experiment (Researcher has full control of the situation)

To run this study as an observational study, a researcher would watch a group of students to see what kind of music they listen to while studying and then compare their grades.

- **Describe how a possible control group would look.**

A control group would be a group that does not listen to any music while studying

Based on the following probability distribution table, determine the mean and standard deviation

X	-5	-2	3	7	11
P(X)	0.3	0.29	0.2	0.16	P(X=11)

(Sum of Probabilities must add to 1.0)

→ 0.05

```
> assign("p",c(.3,.29,.2,.16))
> 1-sum(p)
[1] 0.05
> assign("p",c(.3,.29,.2,.16,0.05))
> sum(p)
[1] 1
> assign("x",c(-5,-2,3,7,11))
> sum(x*p)
[1] 0.19
> sum(x^2*p)-sum(x*p)^2
[1] 24.3139
> sqrt(24.3139)
[1] 4.930913
```

→ Mean: 0.19

→ std dev: 4.931

In testing a study technique, researchers found that 15% of all students using it will have increased scores. A random sample of 25 students using the technique is selected. Find the probability that exactly three will have increased scores. $n = 25, p = .15, x = 3$

```
dbinom(3,25,.15)  
[1] 0.2173792
```

A batter has an RBI of 0.16. We want to observe this batter during one game to see how many at-bats he has before his first home run. What is the probability that the batter's first homerun is on his third time to the plate?

$x=3$

Geometric

```
dgeom(3-1,0.16)  
1] 0.112896
```

The length of time needed to drive to school is normally distributed with mean 32 minutes and standard deviation 7 minutes. Find the probability that it will take less than 27 minutes to drive to school.

```
pnorm(27,32,7)  
[1] 0.2375253
```

Find a value of c so that $P(Z > c) = 0.23$.

1 -

↳ Given probability

```
qnorm(1-0.23)  
[1] 0.7388468
```

In a large population, 72% of the households have multiple cars. A simple random sample of 150 households is to be contacted and the sample proportion computed. What is the mean and standard deviation of the sampling distribution of the sample proportions?

$$\mu_{\hat{p}} = P = 0.72$$

$$\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \text{sqrt}(.72*(1-.72)/150)$$

1] 0.03666061

In a large population, 72% of the households have multiple cars. A simple random sample of 150 households is to be contacted and the sample proportion computed. What is the probability that the sample distribution has a proportion less than 70%?

```
pnorm(0.70,0.72,sqrt(.72*(1-.72)/150))  
.] 0.2926895
```

Determine the correlation coefficient of the following data:

x	2	4	6	8	10
y	12	17	18	22	24

```
> assign("x",c(2,4,6,8,10))  
> assign("y",c(12,17,18,22,24))  
> cor(x,y)  
[1] 0.9820639
```


Determine the LSRL of the data:

x	2	4	6	8	10
y	12	17	18	22	24

```
> lm(y~x)
```

```
Call:
```

```
lm(formula = y ~ x)
```

```
Coefficients:
```

```
(Intercept)  
9.90
```

```
x  
1.45
```

$$\hat{y} = 1.45x + 9.90$$

$$\hat{y} = 9.90 + 1.45x$$

Determine the residual of the data for the value of $x = 6$.

x	2	4	6	8	10
y	12	17	18	22	24

$$y - \hat{y}$$

$$18 - (1.45(6) + 9.90)$$

$$18 - (1.45 * 6 + 9.90)$$

$$[-] -0.6$$

A simple random sample of 100 kindergarteners indicated that 71% of them have imaginary friends. Find the 95% confidence interval that estimates the proportion of them that have imaginary friends. $\hat{p} = .71$ $n = 100$ $CL = 95\%$

```
> .71 - qnorm(1.95/2) * sqrt(.71 * (1 - .71) / 100)
[1] 0.6210643
> .71 + qnorm(1.95/2) * sqrt(.71 * (1 - .71) / 100)
[1] 0.7989357
```

$[.621, .799]$

z-test

The height of 8th graders is known to have a standard deviation of 4 inches. [A simple random sample of 81 of them is chosen and found to have a mean height of 52 inches. Construct a 89% confidence interval for the mean height of 8th grade students.

```
52-qnorm(1.89/2)*4/sqrt(81)  
1] 51.28969  
52+qnorm(1.89/2)*4/sqrt(81)  
1] 52.71031  
.
```

[51.29, 52.71]

A 95% confidence interval for a sample mean is to be constructed, based on having a preliminary standard deviation of 6.4, and a margin of error of less than 3.8. Determine the minimum sample size needed to construct this interval.

$$\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$ME = z^* \frac{\sigma}{\sqrt{n}}$$

$$\frac{ME}{1} = \frac{z^* \sigma}{\sqrt{n}}$$

$$\sqrt{n} \cdot ME = z^* \sigma$$

$$\sqrt{n} = \frac{z^* \sigma}{ME}$$

$$n = \left(\frac{z^* \sigma}{ME} \right)^2$$

```
[1] .....  
(qnorm(1.95/2)*6.4/3.8)^2  
[1] 10.89655
```

$$n = 11$$

The mean final exam score for an English class of 264 students was 78.3 with a standard deviation of 5.3. The mean final exam score for a chemistry class of 127 students was 73.8 with a standard deviation of 1.3. Test the claim that the test scores are approximately equal with a significance of 2%. α (t-test)

$$H_0: \mu_1 = \mu_2$$

$$H_a: \mu_1 \neq \mu_2$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{78.3 - 73.8}{\sqrt{\frac{5.3^2}{264} + \frac{1.3^2}{127}}}$$

```
> (78.3-73.8)/sqrt(5.3^2/264+1.3^2/127)
[1] 13.00618  $\rightarrow$  t test statistic
> 2*pt(-13.00618,126)
[1] 4.774585e-25  $\neq 0 \rightarrow$  p-value
```

$p < \alpha$ R H₀