

MATH 3307

Lesson 10

Random Variables

A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.

A **discrete random variable** is one that can assume a countable number of possible values. A **continuous random variable** can assume any value in an interval on the number line.

A **probability distribution table of X** consists of all possible values of a discrete random variable with their corresponding probabilities.

Example:

Suppose a family has 3 children. Show all possible gender combinations:

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Suppose a family has 3 children. Show all possible gender combinations: *Keep in mind, there will be $(2)(2)(2)=8$ combinations.*

BBB

BGB

GBB

GGB

BBG

BGG

GBG

GGG

Example:

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

Example:

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

We need to know the total possible outcomes. *8 outcomes*

We need to categorize them by number of girls. *0, 1, 2, 3*

We need a probability of each outcome. *Next slide*

Example:

BBB	BGB	GBB	GGB
BBG	BGG	GBG	GGG

Suppose a family has 3 children. Now suppose we want the probability distribution for the number of girls in the family.

Draw a probability distribution table for this example.

0 girls: $\frac{1}{8}$

1 girl: $\frac{3}{8}$

2 girls: $\frac{3}{8}$

3 girls: $\frac{1}{8}$

x	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

The sum of the p-values must equal 1
 $(\frac{1}{8}) + (\frac{3}{8}) + (\frac{3}{8}) + (\frac{1}{8}) = \frac{8}{8} = 1$

Example:

x	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Find the following probabilities:

Find $P(X > 2) = P(x=3)$
 $\frac{1}{8}$

$P(X < 1) = P(x=0)$
 $\frac{1}{8}$

$P(1 < X \leq 3)$

0 girls:

1 girl:

2 girls:

3 girls:

$$\begin{aligned} P(1 < X \leq 3) &= P(x=2) + P(x=3) \\ &= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \end{aligned}$$

Example: Popper # 04

Another example: Suppose you are given the following distribution table:

X	1	2	3	4	5	6	7
P(X)	0.15	0.05	0.10	?	0.10	0.15	0.15

Find the following:

1. $P(X = 4)$

- a. 0.10
- b. 0.15
- c. 0.30
- d. 0.05

2. $P(X < 2) = P(X = 1)$

- a. 0.10
- b. 0.15
- c. 0.20
- d. 0.05

3. $P(2 < X \leq 5)$

- a. 0.50
- b. 0.15
- c. 0.20
- d. 0.25

4. $P(X > 3) =$

- a. 0.50
- b. 0.75
- c. 0.80
- d. 0.70

$$\begin{array}{r}
 P(x=4) \quad .30 \\
 P(x=5) \quad .10 \\
 P(x=6) \quad .15 \\
 P(x=7) \quad .15 \\
 \hline
 .70
 \end{array}$$

$$P(X=3) + P(X=4) + P(X=5) \\
 .10 + .30 + .10$$

$$> 1 - (.15 + .05 + .10 + .10 + .15 + .15) \\
 [1] 0.3$$

Expected Value:

The **mean**, or **expected value**, of a random variable X is found with the following formula:

$$\mu_X = E[X] = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

Multiply each x-value by its corresponding p-value and add those products

What is the expected number of girls in the family above?

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

x	P
0	$1/8$
1	$3/8$
2	$3/8$
3	$1/8$

$$\begin{aligned} E[X] &= 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right) \\ &= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5 \end{aligned}$$

What is the expected number of girls in the family above?

$$\mu_X = E[X] = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

In R Studio: `assign("x",c(values))`
`assign("p",c(probabilities))`
`sum(x*p)`

TI: `x → L1, p → L2,`
`2nd List (STAT), MATH (right arrow), sum (option 5)`
`sum(L1*L2)`

```
> assign("x",c(0,1,2,3))
> assign("p",c(1/8,3/8,3/8,1/8))
> sum(x*p)
[1] 1.5
```

L1	L2	L3
0	.125	---
1	.375	
2	.375	
3	.125	
---	-----	

```
sum(L1*L2)
1.5
```

Variance and Standard Deviation

$$\begin{aligned}\sigma_X^2 &= \text{Var}[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n \\ &= \sum (x_i - \mu_X)^2 p_i\end{aligned}$$

Or (the alternate formula)

$$\sigma_X^2 = \text{Var}[X] = E[\textcircled{X^2}] - (E[X])^2$$

Repeat the Expectancy Formula using x^2 instead of x .

Variance and Standard Deviation

$$\begin{aligned}\sigma_X^2 &= \text{Var}[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n \\ &= \sum (x_i - \mu_X)^2 p_i\end{aligned}$$

In R Studio: `sum((x-mean)^2*p)`

or

~~*~~ `sum(x^2*p)-sum(x*p)^2`

~~*~~ In TI: `sum(L1^2*L2)-sum(L1*L2)^2`

Find the **standard deviation** for the number of girls in the example above

```
- -  
> sum(x^2*p)-sum(x*p)^2  
[1] 0.75  
> sqrt(0.75)  
[1] 0.8660254
```

```
sum(L1^2*L2)-sum(L1)^2  
      .75  
√Ans  
      .8660254038
```


Popper 04 continued:

X	1	2	3	4	5	6	7
P(X)	0.15	0.05	0.10	?	0.10	0.15	0.15

5.

What is the expected value?

(Mean)

- a. 3.50
 b. 4.20
 c. 0.35
 d. 5.00

.30

6,
7.

The variance (~~2~~⁶) and standard deviation (~~2~~⁷)?

- #6 a. 3.66
 b. 13.39
 #7 c. 1.91
 d. 1.55

```
> assign("x",c(1,2,3,4,5,6,7))
> assign("p",c(.15,.05,.10,.30,.10,.15,.15))
> sum(x*p)
[1] 4.2
> sum(x^2*p)-sum(x*p)^2
[1] 3.66
> sqrt(3.66)
[1] 1.913113
```

A carnival game costs \$10 to play. First prize (probability of 0.02) is \$100. Second prize (probability of 0.05) is \$50. Third prize (probability of 0.10) is \$1. What are your expected winnings?

x	90	40	-9	-10
P	.02	.05	.10	.83

(100-10)

.17

```
> assign("x",c(90,40,-9,-10))
> assign("p",c(.02,.05,.10,.83))
> sum(x*p)
[1] -5.4
```

Rules for means and variances:

Suppose X is a random variable and we define W as a new random variable such that $W = aX + b$, where a and b are real numbers. We can find the mean and variance of W with the following formula:

$$E[W] = E[aX + b] = aE[X] + b$$

$$\sigma_W^2 = \text{Var}[W] = \text{Var}[aX + b] = a^2 \text{Var}[X]$$

$$\sigma_w = a[\sigma_x]$$

only use
multiplication
portion
of formula

Popper 04, continued:

$$\mu_x = 22$$
$$\sigma_x = 3$$

Suppose you have a distribution, X , with mean = 22 and standard deviation = 3. Define a new random variable $Y = 3X + 1$.

- 8, Find the variance of X . $(\sigma^2 = 3^2 = 9)$ [choice D]
- 9, Find the mean of Y . $Y = 3(22) + 1 = 66 + 1 = 67$ [choice B]
- 10, Find the variance of Y . $(\sigma^2 = 9^2 = 81)$ [choice E]
11. Find the standard deviation of Y . $Y = 3(3) = 9$ [choice D]
multiply only

Choices for above questions:

- a. 3 b. 67 c. 63 d. 9 e. 81

Use the following Probability Distribution Table to find the values of A, B and C.

X	0	1	2	3	4	5
P(X)	0.15	0.25	0.05	A	B	C

$$A = 0.10$$

$$B = 0.20$$

$$C = 0.25$$

Additional Information: $P(X < 4) = 0.55$; $E[X] = 2.7$

$$P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0.55$$

$$P(X < 4) = 0.15 + 0.25 + 0.05 + A = 0.55$$

$$0.45 + A = 0.55$$

$$A = 0.10$$

$$E[X] = 0(\cancel{0.15}) + 1(0.25) + 2(0.05) + 3(0.10) + 4B + 5C = 2.7$$

$$0.25 + 0.10 + 0.30 + 4B + 5C = 2.7$$

$$0.65 + 4B + 5C = 2.7$$

$$4B + 5C = 2.05$$

$$4B + 5C = 2.05$$

$$\frac{5C}{5} = \frac{2.05 - 4B}{5}$$

$$C = .41 - .8B$$

$$C = .41 - .8(.2)$$

$$C = .41 - .16$$

$$\rightarrow C = 0.41 - 0.8B$$

$$\boxed{C = .25}$$

Sum of all Probabilities is 1.0

$$P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) = 1.0$$

$$.15 + .25 + .05 + .10 + B + 0.41 - 0.8B = 1.0$$

$$0.96 + 1.0B - 0.8B = 1.0$$

$$0.96 + 0.2B = 1.0$$

$$0.2B = 0.04$$

$$\boxed{B = 0.2}$$