

MATH 3307

Lesson 11

# Bernoulli Trials

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only **two options** and each is mutually exclusive).
2. The probability of success is  $p$  and probability of failure is  $q = 1 - p$ .

A **Bernoulli random variable** is a variable assigned to **represent the successes** in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable**.

# Bernoulli Experiment

A **binomial experiment** occurs when the following conditions are met:

1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials.  $(n)$
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.  $(p)$

The random variable  $X$  = number of successes of a binomial experiment is a **binomial distribution** with parameters  $p$  and  $n$  where  $p$  represents the probability of a success and  $n$  is the number of trials. The possible values of  $X$  are whole numbers that range from 0 to  $n$ . As an abbreviation, we say  $X \sim B(n, p)$ .

$$0 \leq X \leq n$$

$$X \sim B(n, p)$$

$$5 \sim B(7, 0.24)$$

This is the probability of 5 successes out of 7 trials, if the probability of success of each trial is 0.24

# Binomial Distribution Formula

**Binomial probabilities** are calculated with the following formula:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = {}_n C_k p^k (1-p)^{n-k}$$

**In R**,  $P(X = k) = \underline{\text{dbinom}(k, n, p)}$ . With a TI-83/84 calculator,  $P(X = k) = \underline{\text{binompdf}(n, p, k)}$

These are used to calculate an exact number of successes in a binomial experiment.

## Example:

$p = 0.50$   $n = 30$

A fair coin is flipped 30 times.

What is the probability that the coin comes up heads exactly 12 times?

$(x, n, p)$   
· `dbinom(12, 30, 0.50)`  
[1] 0.08055309

```
0: quit DRAW  
6: tcdf(  
7: X^2Pdf(  
8: X^2cdf(  
9: Fpdf(  
0: Fcdf(  
1: binomPdf(  
2: binomcdf(  
3: 
```

```
binomPdf  
trials: 30  
p: .5  
x value: 12  
Paste
```

$x = 12$

```
binomPdf(30, .5, 12)  
.080553093
```

$$P(X \leq k) = \text{pbinom}(k, n, p)$$

$$P(X \leq k) = \text{binomcdf}(n, p, k)$$

What is the probability the coin comes up heads less than 12 times?

$X = 0, 1, 2, 3, \dots, 10, 11$

largest  $X$   
 $n$   $p$

```
0: Fcdf(  
A: binompdf(  
3: binomcdf(  
C: poissonpdf(  
)
```

```
binomcdf  
trials: 30  
P: .5  
x value: 11
```

```
> pbinom(11, 30, 0.50)  
[1] 0.1002442
```

Note: "at most 12 times" means that  $x = 0, 1, 2, 3, \dots, 10, 11, 12$

```
binomcdf(30, .5, 12)  
.1002442112
```

What is the probability the coin comes up heads more than 12 times?

$X = 13, 14, 15, 16, \dots, 29, 30$

Note: "at least 12 times" means that  $x = 12, 13, 14, 15, \dots, 30$

$X = 0, 1, 2, \dots, 30$   
throw away  $X = 0, 1, \dots, 12$

```
> 1 - pbinom(12, 30, 0.50)  
[1] 0.8192027
```

```
1 - binomcdf(30, .5, 12)  
.8192026942
```

Leaving  $X = 13, 14, 15, \dots, 30$

# Mean and Variance

The mean and variance of a binomial distribution are computed using the following formulas:

$$\mu = E[X] = np$$

$$\sigma^2 = np(1-p)$$

```
μ = > 30*0.50  
[1] 15  
σ2 = > 30*.50*(1-.50)  
[1] 7.5  
σ = > sqrt(7.5)  
[1] 2.738613
```

The mean represents the x-value with the largest probability associated with it.  
(Or the most likely to occur x-value)



# Popper 05:

From text:

$$p = 0.80$$

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

1. No one will contract the flu?  $x=0$  B
2. All will contract the flu?  $x=5$  D
3. Exactly two will get the flu?  $x=2$  A
4. At least two will get the flu?  $x=2,3,4,5$  C

$$n = 5$$

Choices for all above questions:

- a. 0.0512
- b. 0.00032
- c. 0.99328
- d. 0.32768

```
> dbinom(0,5,0.80)
[1] 0.00032
> dbinom(5,5,0.80)
[1] 0.32768
> dbinom(2,5,0.80)
[1] 0.0512
> 1-pbinom(1,5,0.80)
[1] 0.99328
```

## Example...Continued

From text:

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

Let  $X$  = number of family members contracting the flu. Create the probability distribution table of  $X$ .

```
> dbinom(1,5,0.80)
[1] 0.0064
> dbinom(3,5,0.80)
[1] 0.2048
> dbinom(4,5,0.80)
[1] 0.4096
```

$X$	0	1	2	3	4	5
$P(X)$	.00032	.0064	.0512	.2048	.4096	.32768

Find the mean and variance of this distribution.

```
> 5*0.80
[1] 4
> 5*.80*(1-.80)
[1] 0.8
> sqrt(0.80)
[1] 0.8944272
```

Mean: 4  
Variance: 0.8  
Standard Deviation: 0.8944