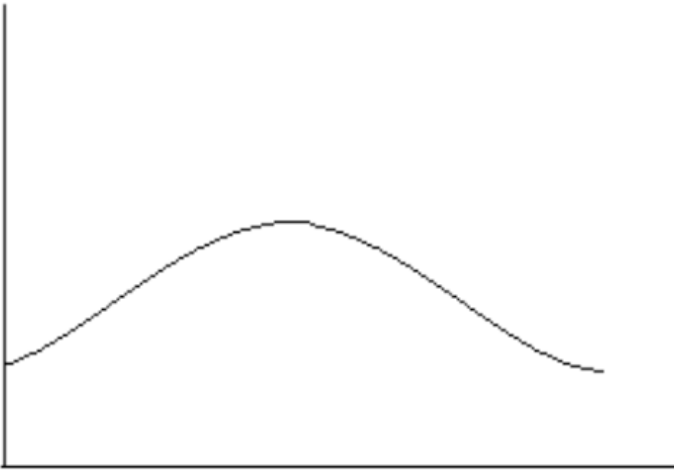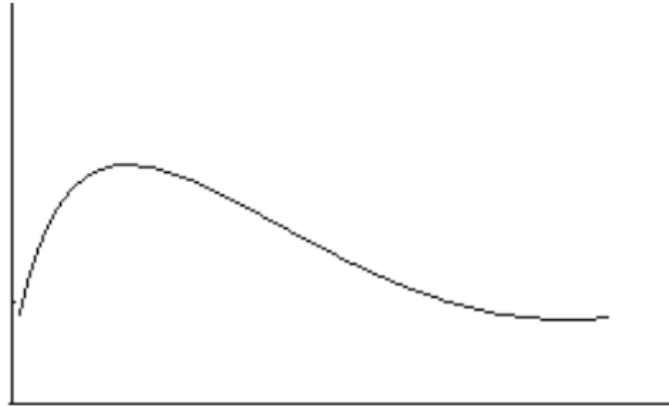# MATH 3307
Lesson 13

Density Curves

A *density curve* is a graph whose area between it and the x-axis is equal to one. These graphs come is a variety of shapes but the most familiar "normal" graph is bell shaped. The area under the curve in a range of values indicates the proportion of values in that range.

Skewness and curves:
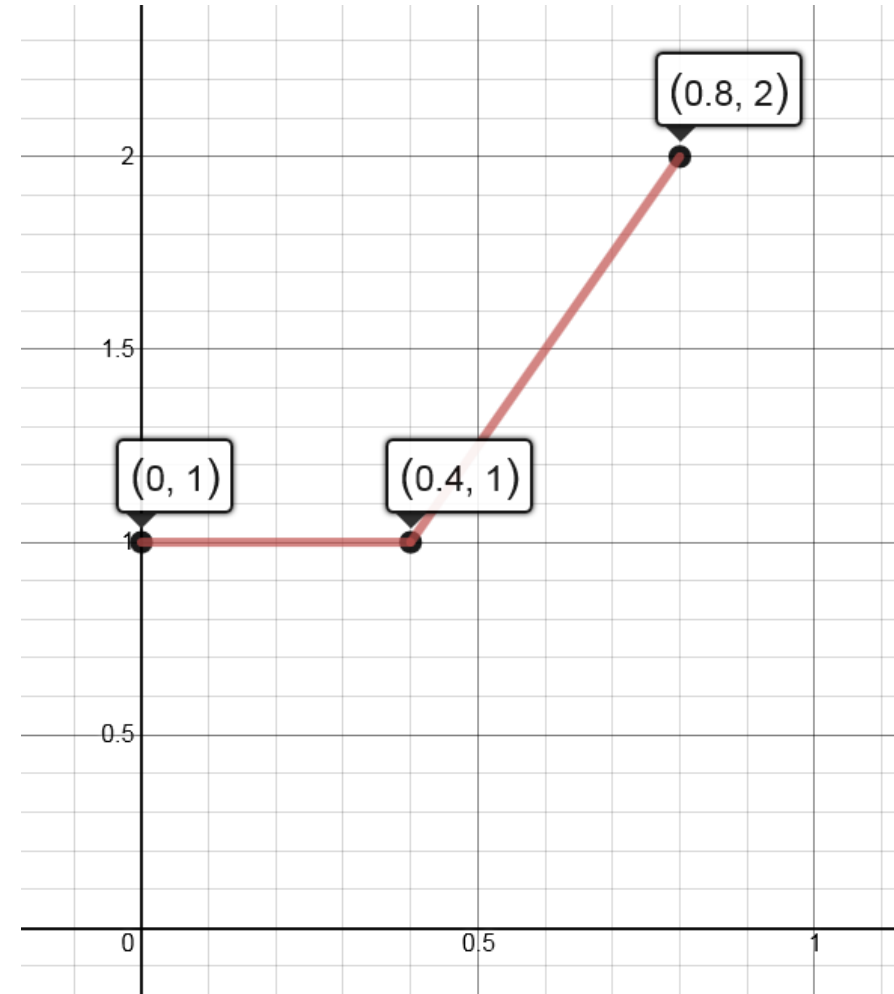
Bell Shaped (normal)

Skewed Right

Skewed Left

Example: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.

Sketch:

Example: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.
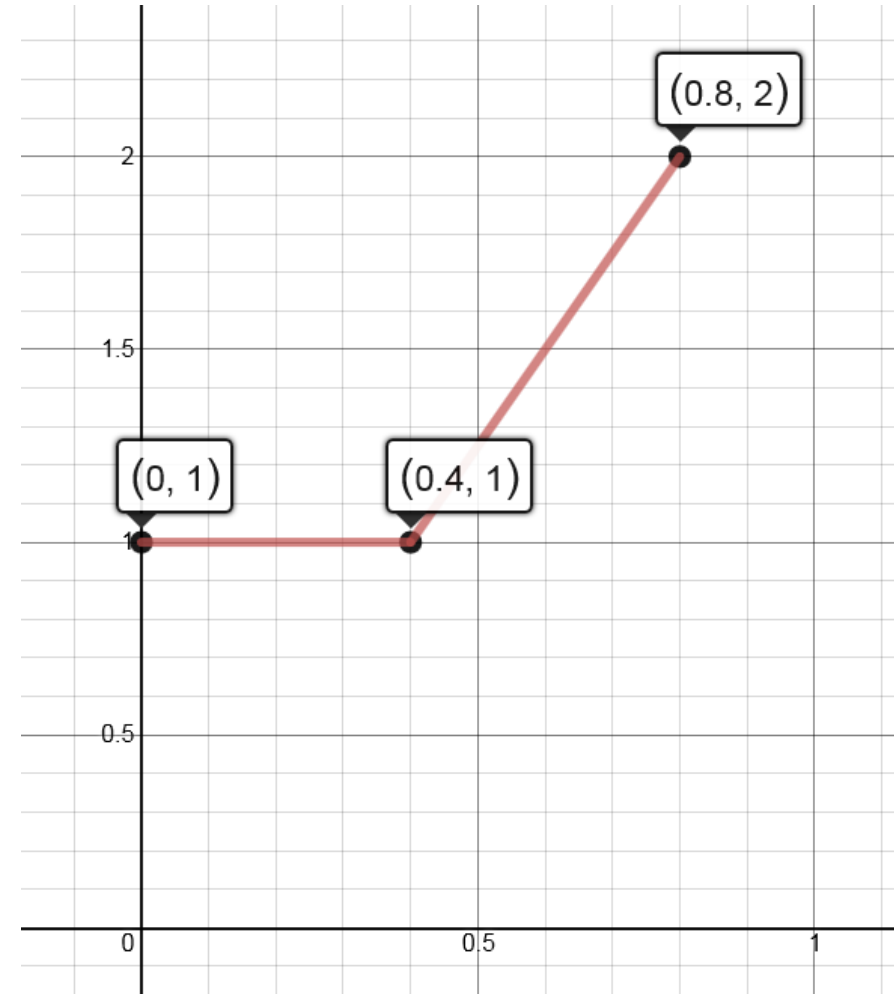
Sketch:

Example: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (.4, 1). The second goes from (.4, 1) to (.8, 2) in the xy plane.

What percent of observations fall below .4?

What percent of observations lie between .4 and .8?

What percent of observations are equal to .4?

Example: Consider a uniform density curve defined from $x = 0$ to $x = 6$.
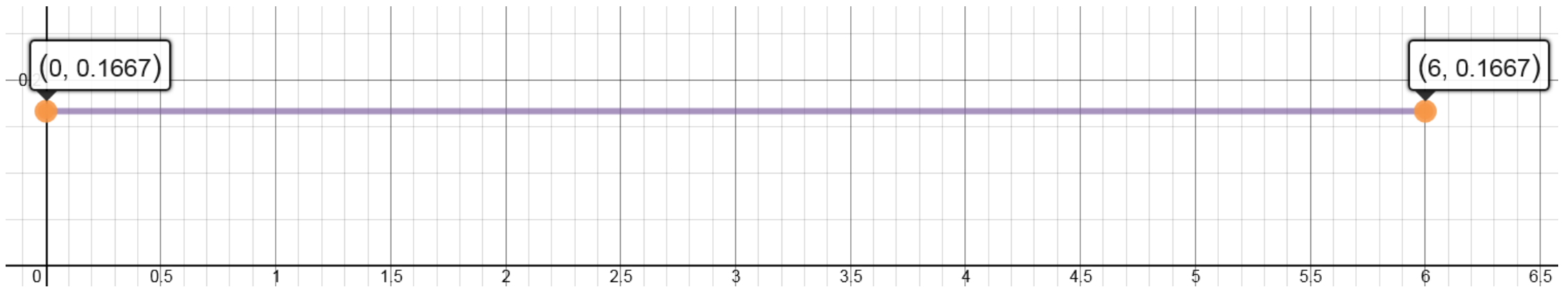Sketch:

What percent of observations fall below 2?

What percent of observations lie between 2 and 3?

Find the median.

Example: Consider a uniform density curve defined from *x* = 0 to *x* = 6.
Sketch:

(0, 0.1667)   (6, 0.1667)

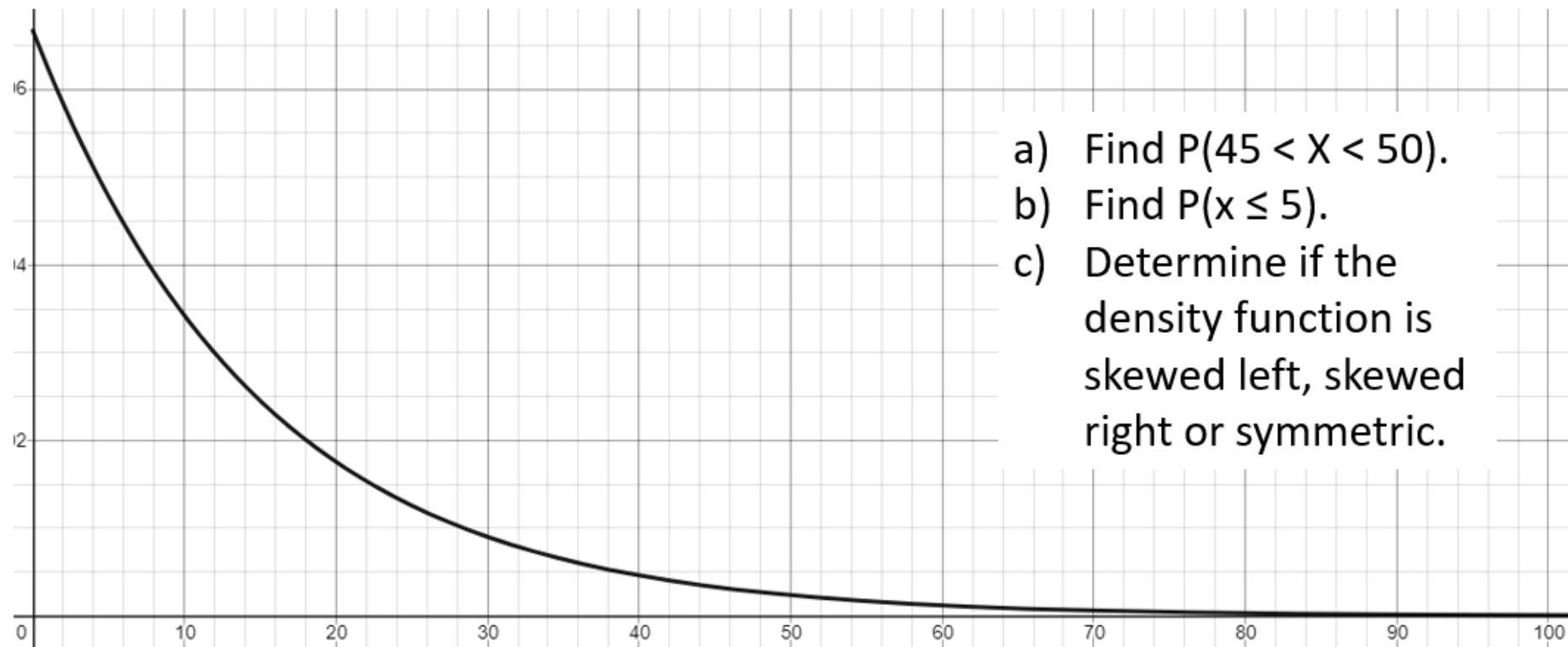What percent of observations fall below 2?

What percent of observations lie between 2 and 3?

Find the median.

Another Example:

A probability density curve consists of two line segments. One segment connects the points (0, 2) and (0.25, 2), and the other connects (0.25, 2) to the x-axis. Determine P(x > 0.35).

The probability density function, illustrated here, is defined for x ≥ 0, and has a mean of x = 15. The area enclosed by this graph and the x-axis between x = 0 and x = a is given by: $1 - e^{(-a/15)}$.

a) Find P(45 < X < 50).
b) Find P(x ≤ 5).
c) Determine if the density function is skewed left, skewed right or symmetric.

The probability distribution function illustrated here is defined for all x-values. The area enclosed by the graph and the x-axis between x = 0 and x = a is given by: $\frac{1}{\pi} \cdot \tan^{-1} \left(\frac{a^3 + 2a^2 + 3a + 5}{5} \right) - \frac{1}{4}$.

Determine, assuming the mode occurs at x = -2.125, is the graph symmetric about that peak? What significance is x = 0 in this function?