MATH 3307
Lesson 31

Inference for the Mean of a Population

In chapter 7, we discussed one type of inference about a population – the confidence interval. In this chapter we will begin discussion the **significance test**.

H_0 : is the **null hypothesis.** The **null hypothesis** states that there is no effect or change in the population. It is the statement being tested in a test of significance.

This is the accepted value that we are trying to confirm or deny

H_a : is the **alternate hypothesis**. The **alternative hypothesis** describes the effect we suspect is true, in other words, it is the alternative to the "no effect" of the null hypothesis.

Three possible Alternate Hypothesis: Null hypothesis is too large, too small, or incorrect.

Since there are only two hypotheses, there are only two possible decisions: *reject the null hypothesis in favor of the alternative* or *don't reject the null hypothesis*. We will never say that we accept the null hypothesis.

RH_0

FRH_0

For inference about a population mean:

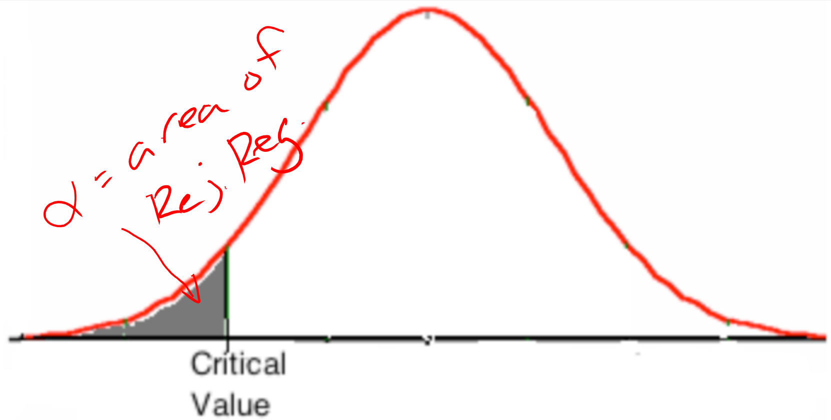
$H_0 : \mu = \mu_0$ where μ_0 represents the given population mean.

$H_0 : \mu = 150$

$\hookrightarrow$ Accepted Value

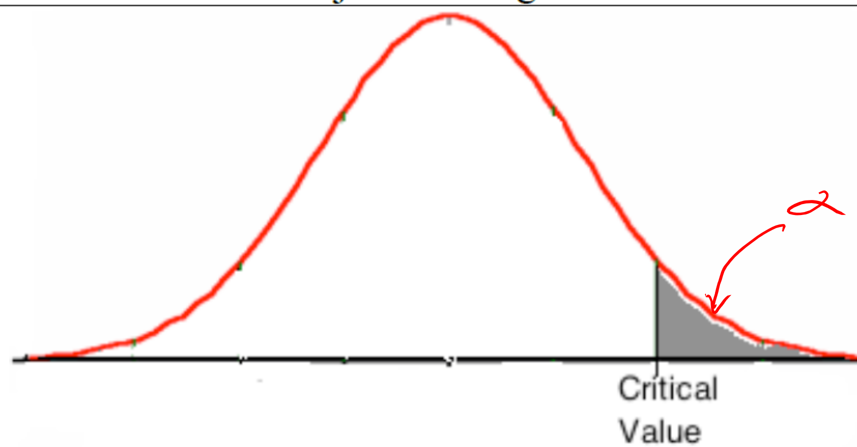
Current Value

For inference about a population mean:

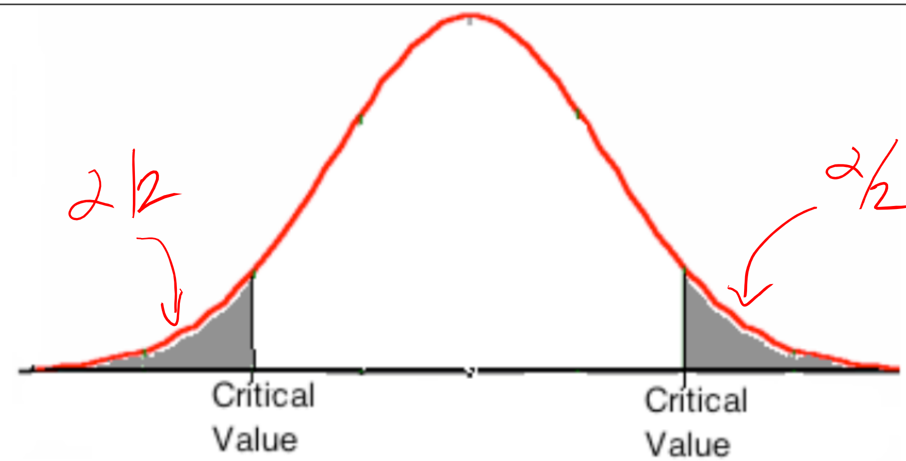
Alternate Hypothesis	Rejection Region
$H_a : \mu < \mu_0$ To find a critical value: qnorm(a) [z test] qt(a,df) [t test]	

If you are given a Significance Level, α, this can be used to determine the critical value and the rejection region. The total area of the rejection region will have the value of α.
The rejection region is the set of values of the test statistic that will lead to a rejection of the null hypothesis.
The critical value is the boundary of the rejection region.

For inference about a population mean:

Alternate Hypothesis	Rejection Region
$H_a : \mu > \mu_0$ To find the critical value: qnorm(1-a) [z test] qt(1-a,df) [t test]	

For inference about a population mean:

Alternate Hypothesis	Rejection Region
$H_a : \mu \neq \mu_0$ To find critical values: ± qnorm(a/2) ± qt(a/2,df)	

The probability, computed assuming that the null hypothesis is true, that the test statistic would take a value as extreme or more extreme than that actually observed is called the ***p*-value** of the test. A result with a small *p*-value is called **statistically significant**. This means that chance alone would rarely produce so extreme a result. We say that a value is **statistically significant** when the *p*-value is as small as, or smaller than, the given significance level, α. If we are not given α, we can interpret the results like this:

- If the *p*-value is less than 1%, we say that there is overwhelming evidence to infer that the alternative hypothesis is true. (We also say that the test is highly significant)
- If the *p*-value is between 1% and 5%, we say that there is strong evidence to infer that the alternative hypothesis is true. (We also say that the test is significant)
- If the *p*-value is between 5% and 10%, we say that there is weak evidence to infer that the alternative hypothesis is true. (We also say that the test not statistically significant)
- If the *p*-value is exceeds 10%, we say that there is no evidence to infer that the alternative hypothesis is true.

To summarize:

if α is given:
- $p < \alpha$ means to RH_o
- $p > \alpha$ means to FRH_o

if α is not given:
- $p < 10\%$ means Rh_o (with varying certainty)
- $p > 10\%$ means FRH_o

To find p-value:
Ha less than: pnorm(z) or pt(t,df)
Ha greater than: 1-pnorm(z) or 1-pt(t,df)
Ha is not equal: 2*pnorm(z) or 2*pt(t,df) {must use negative z,t}

Steps to follow:

When performing a significance test, we follow these steps:

1. Check assumptions.
2. State the null and alternate hypotheses.
3. Graph the rejection region, labeling the critical values.
4. Calculate the test statistic.
5. Find the p-value. If this answer is less than the significance level, α, we can reject the null hypothesis in favor of the alternate.
6. Give your conclusion using the context of the problem. When stating the conclusion you can give results with a confidence of $(1-\alpha)(100)\%$.

Check the casa calendar: Help with Hypothesis testing PDF. This is an interactive PDF that will guide you through all the steps you need to do to complete your hypothesis test.

Z Test (to calculate the test statistic):

z – test

Assumptions:

1. An SRS of size n from the population.
2. Known population standard deviation, σ.
3. Either a normal population or a large sample ($n \geq 30$).

To compute the z – test statistic, we use the formula: $\boxed{z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$.

T Test (to calculate the test statistic):

***t* – test**

Assumptions:

1. An SRS of size n from the population.
2. Unknown population standard deviation.
3. Either a normal population or large sample ($n \geq 30$).

To compute the t – test statistic, we use the formula: $\boxed{t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}}$, where s is the sample standard deviation. The t – test will use $n - 1$ degrees of freedom.

Example:

$H_0: \mu = 18.2$

$H_a: \mu \neq 18.2$

Population:
$\mu = 18.2$
$\sigma = 1.38$

Z-test

Example 1: A laboratory is asked to evaluate the claim that the amount of the active ingredient in a bug spray is 18.2 grams for a 70-gram bottle with a standard deviation of 1.38 grams. The mean amount of the active ingredient in 40 randomly selected 70-gram bottles of the bug spray is $\bar{x} = 16.828$ grams. Do these analyses indicate that the amount of the active ingredient is different than the original claim at an $\alpha = 0.05$ significance level?

Sample: $n = 40$

$\bar{x} = 16.828$


```
qnorm(0.05/2)
[1] -1.959964
```

$\alpha = .05$

Rej. Reg: $z < -1.96$ or $z > 1.96$

Example:

Example 1: A laboratory is asked to evaluate the claim that the amount of the active ingredient in a bug spray is 18.2 grams for a 70-gram bottle with a standard deviation of 1.38 grams. The mean amount of the active ingredient in 40 randomly selected 70-gram bottles of the bug spray is $\bar{x} = 16.828$ grams. Do these analyses indicate that the amount of the active ingredient is different than the original claim at an $\alpha = 0.05$ significance level?

H_0: $\mu = 18.2$
H_a: $\mu \neq 18.2$

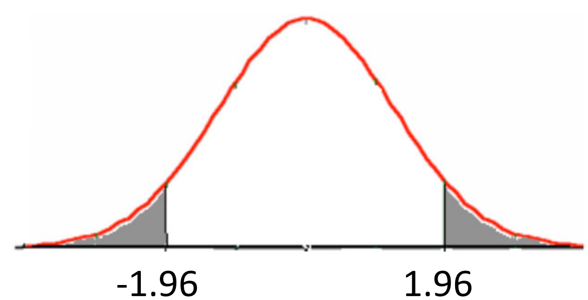
Population:
$\mu = 18.2$
$\sigma = 1.38$

$\bar{x} = 16.828$

$\alpha = 0.05$

Since the population standard deviation is given, use a z-test.

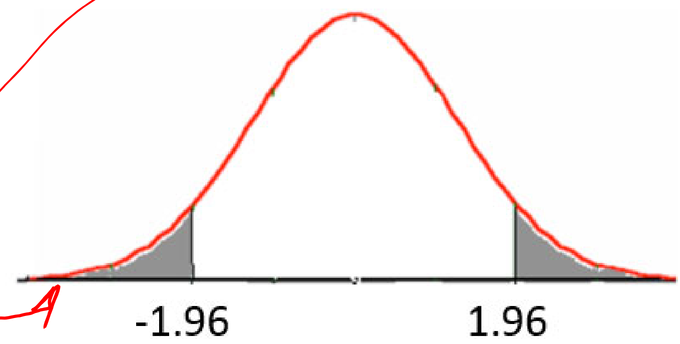
Rejection Region is 2-sides, so the area of one tail is 0.05/2 = 0.025.
qnorm (1-0.025) = 1.96 qnorm(0.025) = 1.96



Example (Continued):

```
(16.828-18.2)/(1.38/sqrt(40))
] -6.287891
```

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{16.828 - 18.2}{1.38/\sqrt{40}} = -6.288$$



z = -6.288 falls within our rejection region

(-6.288 < -1.96) $R\ H_0$

```
2*pnorm(-6.287891)
1] 3.218077e-10
```

P-Value: P(Z<-6.288)+P(Z>6.288)=2P(Z<-6.288) ≈ 0 $P = 0 < \alpha = .05$

(Since p < α, we can conclude that we can reject the null hypothesis.)

$(100-\alpha)$

Conclusion: Based on 95% certainty, we can reject the null hypothesis, in favor of saying that the amount of active ingredient is not 18.2 grams per can.

Example: Popper 27

$\mu = 200$ $\sigma = 9$ $\Big\}$ z-test

Mr. Murphy is an avid golfer. Suppose he has been using the same golf clubs for quite some time. Based on this experience, he knows that his average distance when hitting a ball with his current driver (the longest-hitting club) under ideal conditions is 200 yards with a standard deviation of 9. After some preliminary swings with a new driver, he obtained the following sample of driving distances:

205	198	220	210	194	201	213	191	211	203

$n = 10$

He feels that the new club does a better job. Do you agree?

$H_0: \mu = 200$

$H_a: \mu > 200$

```
assign("x",c(205,198,220,210,194,201,213,191,211,203))
mean(x)
1] 204.6
```

1. What is the sample mean?

a. 200 b. 204.6 c. 190 d. 215

2. What is the population mean? Given

a. 200 b. 204.6 c. 190 d. 215

3. Which test statistic can be used?

a. z-test b. t-test

4. What is our null hypothesis?

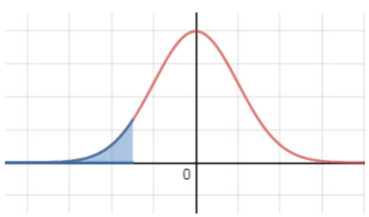
a. $\mu = 200$ b. $\mu < 200$ c. $\mu > 200$ d. $\mu \neq 200$

5. What is our alternate hypothesis?

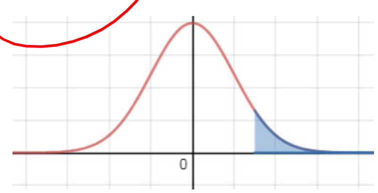
a. $\mu = 200$ b. $\mu < 200$ c. $\mu > 200$ d. $\mu \neq 200$

6. Which of the following is a correct representation of our rejection region?

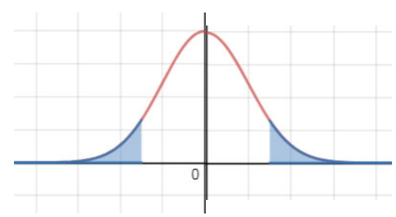
a.



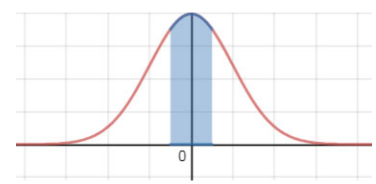
b.



c.



d.



Since we were not provided a significance level (alpha) in this question, we cannot provide a numeric rejection region. Our conclusions will be based entirely on the p-value.

7. What is the value of the test statistic?

$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

```
(mean(x)-200)/(9/sqrt(10))
] 1.616275
```

a. 5.11 b. 4.85 c. -1.616 d. 1.616

8. What is the p-value?

```
1-pnorm(1.616275)
] 0.05301743
```

a. 0.0530 b. 0.9474 c. 0.016 d. 0.9804

9. Do we reject the null hypothesis in favor of the alternate hypothesis?

a. Yes, with overwhelming evidence.
b. Yes, with strong evidence.
c. Yes, with weak evidence.
d. No, we fail to reject the null hypothesis.

$P = 5.3\%$

$.05 < P < .10$

RH_0 (Weak Evidence)