MATH 3307
Lesson 6

Counting Techniques

Combinatorics is the study of the number of ways a set of objects can be arranged, combined, or chosen; or the number of ways a succession of events can occur. Each result is called an **outcome**. An **event** is a subset of outcomes. When several events occur together, we have a **compound event**.

Outcome: all ways that something can happen.
Event: specific conditions about how something can happen.

The Fundamental Counting Principle

The **Fundamental Counting Principle** states that the total number of a ways a compound event may occur is $n_1 \cdot n_2 \cdot n_3 \cdot \ldots\ldots \cdot n_i$ where n_1 represents the number of ways the first event may occur, n_2 represents the number of ways the second event may occur, and so on.

Example:

You are not allowed repeats (such as “double mushrooms”)

Example:

How many ways can you create a pizza choosing a meat and two veggies if you have 3 choices of meats and 4 choices for veggies?

$$\underset{\text{Meat}}{\underline{3}} \times \underset{\text{Veg 1}}{\underline{4}} \times \underset{\text{Veg 2}}{\underline{3}} = 36 \text{ options}$$

(one of the four was used in step, 3 options remain)

Rstudio and TI Commands:

Rstudio:

factorials: (such as 5!)

factorial(5)

combinations: (such as $_6C_2$)

choose(6,2)

(No permutation for Rstudio)

TI 83/84:

Select MATH → PRB (right arrow)

factorial: (option 4)

5!

permutation: (option 2)

$_6P_2$ will be written as 6 nPr 2

combination: (option 3)

$_6C_2$ will be written as 6 nCr 2

Permutations

A **permutation** of a set of *n* objects is an ordered arrangement of the objects.

$${}_nP_n = n(n-1)(n-2)....3\cdot 2\cdot 1 = n!$$

All objects are placed in order

$${}_nP_r = \frac{n!}{(n-r)!}$$

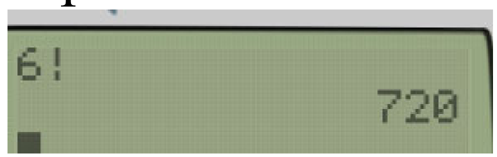
Some of the objects are placed in order

Examples:

How many ways can six people be seated in a row? All six are being seated (factorial)

$6!$

```
> factorial(6)
[1] 720
```

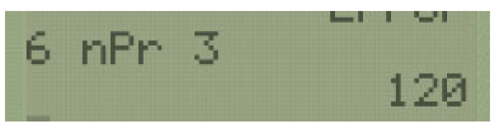


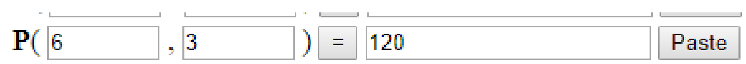
720

In how many ways can 3 of the six symbols, &^%$#@ be arranged?

Since we are only arranging 3 out of the 6 options, this is a permutation.

$_6P_3$





120

With Repetition

When we allow repeated values, The number of orderings of n objects taken r at a time, with repetition is n^r. Example:

n: number of items we are selecting from r: number of selections being made

In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

$n = 6$

$r = 4$

$6^4 = 1296$

(n^r)

Duplicate Objects

The number of permutations, P, of *n* objects taken n at a time with *r* objects alike, *s* of another kind alike, and *t* of another kind alike is

$$P = \frac{n!}{r!s!t!}$$

r, s, t, etc are the individual categories
n is the total number of items to select from
(r+s+t+….=n)

Example:

How many different words (they do not have to be real words) can be formed from the letters in the word MISSISSIPPI?

$$P = \frac{n!}{r!s!t!}$$

M : 1

I : 4

S : 4

P : 2

Total : 11

$$P = \frac{11!}{\cancel{(1!)}(4!)(4!)(2!)}$$

note: $1! = 1$

```
> factorial(11)/(factorial(4)*factorial(4)*factorial(2))
[1] 34650
```

Circular Permutations

The number of circular permutations of n objects is $(n - 1)!$

Example:

In how many ways can 12 people be seated around a circular table?

$(12-1)!$

```
> factorial(12-1)
[1] 39916800
> factorial(11)
[1] 39916800
```

Combinations:

Order does not matter
Treatment of each selection is the same.

A **combination** gives the number of ways of picking *r* unordered outcomes from *n* possibilities. The number of combinations of a set of *n* objects taken *r* at a time is

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Example:

Creating a smaller group from a larger group: combination

How many ways can a committee of 5 be chosen from a group of 12 people?

$_{12}C_5$

```
> choose(12,5)
[1] 792
```

```
12 nCr 5
                792
```

792

C(12 , 5) = 792 Paste

Permutation or Combination:

How many ways can 1^{st}, 2^{nd}, and 3^{rd} place ribbons be awarded when there are 15 contestants? Different treatment of selections: permutation

P(15 , 3) = 2730

How many ways can you be dealt a poker hand of 7 cards from a standard deck of 52? No special treatment for different selections: combination

C(52 , 7) = 133784560 Paste

How many ways can a class President, Vice President, Secretary, Treasurer, and Historian be selected from a class of 500?

Each selection has a different treatment: permutation

P(500 , 5) = 30629362512000 Paste