# MATH 3307
# Lesson 6

# Counting Techniques

**Combinatorics** is the study of the number of ways a set of objects can be arranged, combined, or chosen; or the number of ways a succession of events can occur. Each result is called an **outcome**. An **event** is a subset of outcomes. When several events occur together, we have a **compound event**.

# The Fundamental Counting Principle

The **Fundamental Counting Principle** states that the total number of a ways a compound event may occur is $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_i$ where $n_1$ represents the number of ways the first event may occur, $n_2$ represents the number of ways the second event may occur, and so on.

# Example:

Example:

How many ways can you create a pizza choosing a meat and two veggies if you have 3 choices of meats and 4 choices for veggies?

# Rstudio and TI Commands:

Rstudio:

factorials: (such as 5!)

factorial(5)

combinations: (such as ${}_6C_2$)

choose(6,2)

TI 83/84:

Select MATH → PRB (right arrow)

factorial: (option 4)

5!

permutation: (option 2)

${}_6P_2$ will be written as 6 nPr 2

combination: (option 3)

${}_6C_2$ will be written as 6 nCr 2

# Permutations

A **permutation** of a set of $n$ objects is an ordered arrangement of the objects.

$${}_nP_n = n(n-1)(n-2)....3 \cdot 2 \cdot 1 = n!$$

All objects are placed in order

$${}_nP_r = \frac{n!}{(n-r)!}$$

Some of the objects are placed in order

# Examples:

How many ways can six people be seated in a row?

In how many ways can 3 of the six symbols, &^%$#@ be arranged?

# With Repetition

When we allow repeated values, The number of orderings of n objects taken r at a time, with repetition is $n^r$ . Example:

In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

# Duplicate Objects

The number of permutations, P, of *n* objects taken n at a time with *r* objects alike, *s* of another kind alike, and *t* of another kind alike is

$$P = \frac{n!}{r!s!t!}$$

# Example:

How many different words (they do not have to be real words) can be formed from the letters in the word MISSISSIPPI?

$$P = \frac{n!}{r!s!t!}$$

# Circular Permutations

The number of circular permutations of n objects is $(n - 1)!$

Example:

In how many ways can 12 people be seated around a circular table?

# Combinations:

A **combination** gives the number of ways of picking *r* unordered outcomes from *n* possibilities. The number of combinations of a set of *n* objects taken *r* at a time is

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

# Example:

How many ways can a committee of 5 be chosen from a group of 12 people?

# Permutation or Combination:

How many ways can $1^{st}$, $2^{nd}$, and $3^{rd}$ place ribbons be awarded when there are 15 contestants?

How many ways can you be dealt a poker hand of 7 cards from a standard deck of 52?

How many ways can a class President, Vice President, Secretary, Treasurer, and Historian be selected from a class of 500?