MATH 3307
Lesson 8

Basic Probability Models

- The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

  This is a measure of how likely an event is to occur.
  Example: How likely is it to roll a 4 on a 6-sided die

- The **sample space** of a random phenomenon is the set of all possible outcomes.

  All possible numbers that can turn up:
  S = {1, 2, 3, 4, 5, 6}

- An **event** is an outcome or a set of outcomes of a random phenomenon. It is a subset of the sample space. A **simple event** is an event consisting of exactly one outcome.

  The desired results (subset of the sample space)
  E = {4}

Computing Probability

- To compute the probability of some event *E* occurring, divide the number of ways that *E* can occur by the number of possible outcomes the sample space, *S*, can occur:

$$P(E) = \frac{n(E)}{n(S)}$$

(number of elements in event)/
(number of elements in sample)

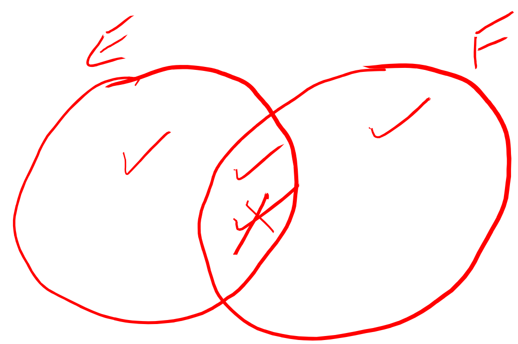
P(4) = 1/6

Basic Rules of Probability

0: Probability of something impossible
1: Probability of something guaranteed

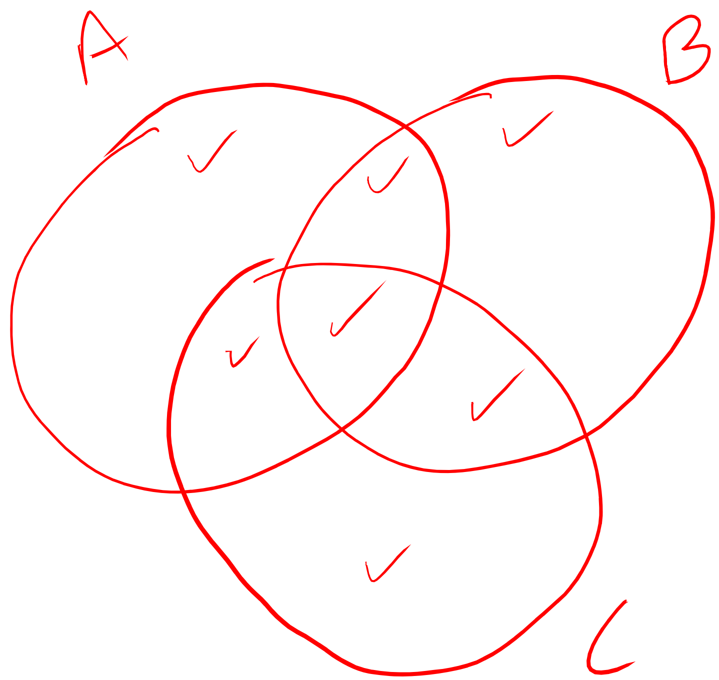
1. All events have a probability between zero and one. $0 \le P(E) \le 1$
2. All possible outcomes together must have a probability of one. $P(S) = 1$
3. Complement Rule: For any event E, $P(E^c) = 1 - P(E)$
4. Addition Rule: If A and B are disjoint events, then $P(E \cup F) = P(E) + P(F)$
5. If E and F are **any** events of an experiment, then $P\left(E \cup F\right) = P\left(E\right) + P\left(F\right) - P\left(E \cap F\right)$

#3: P(Not 4) = 1 – P(4) = 1 – 1/6 = 5/6
#4: P(4 or 5) = P(4) + P(5) = 1/6 + 1/6 = 2/6 = 1/3
#5: P(4 or even) = P(4) + P(even) – P(4 and even) = 1/6 + 3/6 – 1/6 = 3/6 = 1/2



Rule 5a:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$



A deck of cards:

52 total cards

- 26 red cards
  - 13 hearts : A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
  - 13 diamonds: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

- 26 black cards
  - 13 spades: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
  - 13 clubs: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Popper 02:

a. 0 b. 0.5 c. 0.538 d. 0.75 e. 1

You are selecting one card from a standard deck of 52 cards:

1. What is the probability that the card is red or a club?

Rule #4: P(Red or Club) = P(Red) + P(Club) = 26/52 + 13/52 = 0.75 Choice D

2. What is the probability that the card is a queen or a black card?

Rule #5: P(Queen or Black) = P(Q) + P(B) – P(Q and B) = 4/52 + 26/52 – 2/52 = 0.538 Choice C

3. What is the probability that the card is an emperor?

Rule #1: P(E) = 0 *There is no emperor card in a deck of cards* Choice A

4. What is the probability that the card is not a spade?

Rule #3: P(Not S) = 1 – P(S) = 1 – 13/52 = 0.75 Choice D

5. What is the probability that the card is a diamond, heart, spade or club?

Rule #2: P(D or H or S or C) = P(Sample) = 1 Choice E

Examples:

If 5 marbles are drawn at random all at once from a bag containing 8 white and 6 black marbles, what is the probability that 2 will be white and 3 will be black?

Sample

Black: 6

White: 8

Total: 14

Event

Black: 3

White: 2

Total: 5

$$\frac{\overset{\text{Black}}{{}_6C_3} \times \overset{\text{White}}{{}_8C_2}}{{}_{14}C_5}$$

Total

```
> (choose(6,3)*choose(8,2))/choose(14,5)
[1] 0.2797203
```

Examples:

Sample	Event(a)	Event(b)
Men: 5	Men: 0	Men: 3
Women: 7	Women: 6	Women: 3
Total: 12	Total: 6	Total: 6

The qualified applicant pool for six management trainee positions consists of seven women and five men.

a. What is the probability that a randomly selected trainee class will consist entirely of women?

$$\frac{{}_5C_0 \times {}_7C_6}{{}_{12}C_6}$$

```
> (choose(5,0)*choose(7,6))/choose(12,6)
[1] 0.007575758
```

b. What is the probability that a randomly selected trainee class will consist of an equal number of men and women?

$$\frac{{}_5C_3 \times {}_7C_3}{{}_{12}C_6}$$

```
> (choose(5,3)*choose(7,3))/choose(12,6)
[1] 0.3787879
```

Examples:

A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.

a. What is the probability that a randomly selected student likes basketball or hockey? Solve this by also using an appropriate formula.

$$P(B \text{ or } H) = P(B) + P(H) - P(B \text{ and } H)$$

$$= .66 + .38 - .22 = .82$$

Examples:

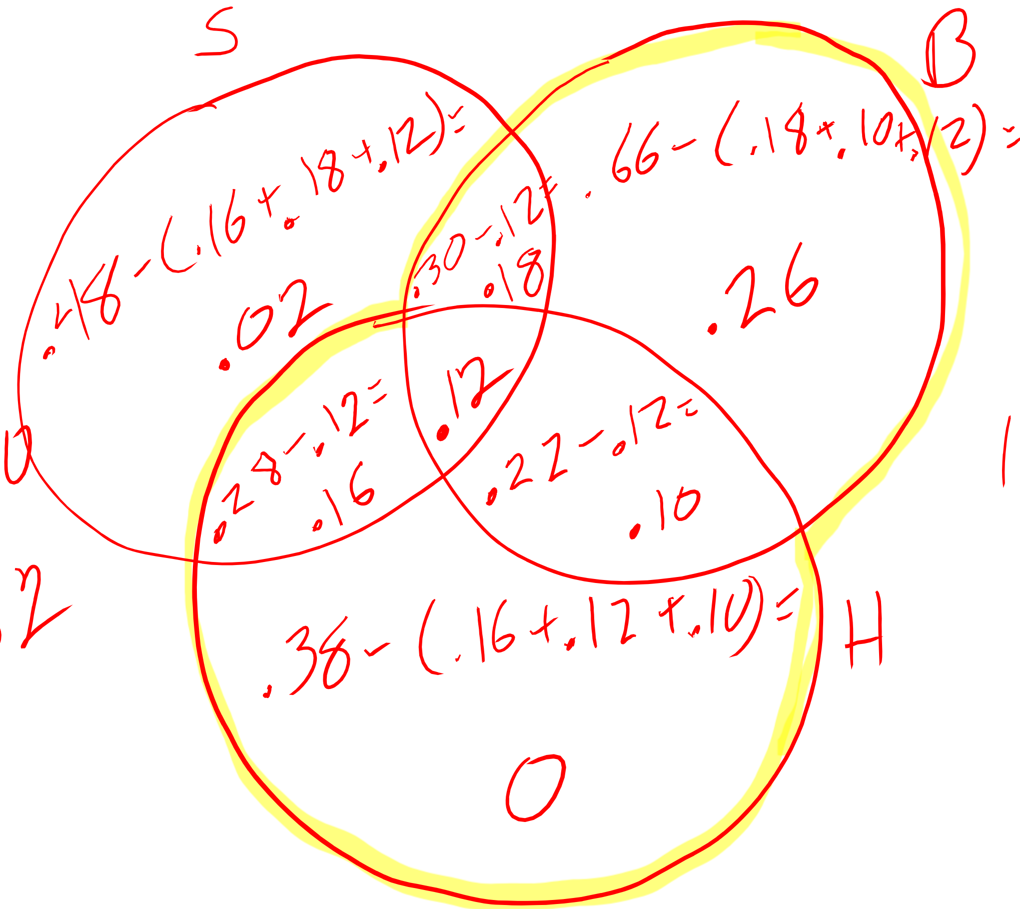
A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.

b. What is the probability that a randomly selected student does not like any of these sports? $1 - P(S \cup B \cup H) = 1 - .84 = \boxed{.16}$

$$P(S \cup B \cup H) = P(S) + P(B) + P(H) - P(S \cap B) - P(S \cap H) - P(S \cap H) + P(S \cap B \cap H)$$

$$= .48 + .66 + .38 - .30 - .22 - .28 + .12 = .84$$

A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.



Question 1:

.16+.12+.26+.10

+.18+0 = .82

1.00-(.02+.18+.26

+.16+.12+.10+0)=

.16

Question 2