

MATH 3307

Lesson 9

General Probability Rules

Two events are **independent** if knowing that one occurs does not change the probability that the other occurs.

(Note: This is not the same as sets that are disjoint or mutually exclusive)

If E and F are independent events, then $P(E \cap F) = P(E)P(F)$

This is also used as a test of independence

Example:

If $P(A) = .36$ and $P(B) = .58$ and A and B are independent, what is $P(A \text{ and } B)$?

$$P(A \cap B) = P(A) \cdot P(B) = (.36) \cdot (.58) = .2088$$

Dependent Events

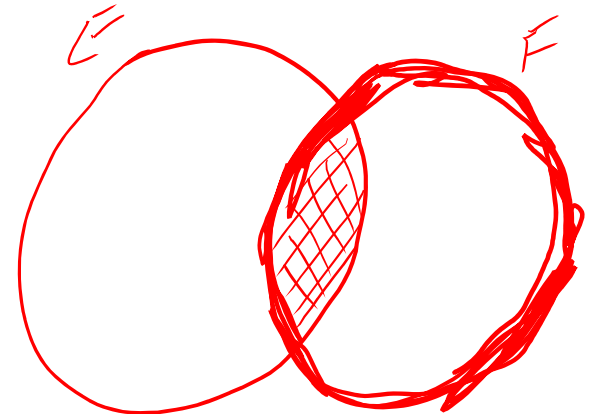
Dependent events, the occurrence of one event does have an effect on the occurrence of the other event. The probability $P(E|F)$ is read “the probability of event E given event F had already occurred”. If E and F are

independent, then $P(E|F) = P(E)$. If events E and F are dependent then

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

This means $P(E \cap F) = P(E|F) \cdot P(F)$

(Cross multiplication)



Examples:

A clothing store that targets young customers (ages 18 through 22) wishes to determine whether the size of the purchase is related to the method of payment. A sample of 300 customers was analyzed and the information is below:

	Cash	Credit	Layaway	Total
Under \$40	60	30	10	100
\$40 or more	40	100	60	200
Total	100	130	70	300

	Cash	Credit	Layaway	Total
Under \$40	60	30	10	100
\$40 or more	40	100	60	200
Total	100	130	70	300

- a. If a customer is selected at random from this group of customers, what is the probability that the customer paid cash?

$$P(\text{Cash}) = \frac{100}{300} = 0.333$$

- b. If a customer is selected at random from this group of customers, what is the probability that the customer paid with a credit card?

$$P(\text{Credit}) = \frac{130}{300} = .433$$

- c. If a customer is selected at random from this group of customers, what is the probability that the customer paid with the layaway plan?

$$P(\text{Layaway}) = \frac{70}{300} = 0.233$$

	Cash	Credit	Layaway	Total
Under \$40	60	30	10	100
\$40 or more	40	100	60	200
Total	100	130	70	300

d. If a customer is selected at random from this group of customers, what is the probability that the customer purchased under \$40?

$$P(< 40) = \frac{100}{300} = .333$$

e. If a customer is selected at random from this group of customers, what is the probability that the customer purchased \$40 or more?

$$P(\geq 40) = \frac{200}{300} = .667$$

	Cash	Credit	Layaway	Total
Under \$40	60	30	10	100
\$40 or more	40	100	60	200
Total	100	130	70	300

$$P(\text{Credit}) = \frac{130}{300}$$

$$P(\text{LA}) = \frac{70}{300}$$

f. If a customer is selected at random from this group of customers, what is the probability that the customer paid with a credit card given that the purchase was under \$40?

$$P(\text{Credit} | <40) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)} = \frac{30}{100} = 0.3 \neq P(\text{Credit})$$

g. If a customer is selected at random from this group of customers, what is the probability that the customer paid with the layaway plan given that the purchase was \$40 or more?

$$P(\text{LA} | \geq 40) = \frac{n(A \cap B)}{n(B)} = \frac{60}{200} = 0.3 \neq P(\text{LA})$$

Popper 03: $P(A) = 0.6$

$P(B) = 0.3$

$P(A \cup B) = 0.8$

1. Determine the value of $P(A \cap B) = .10$

2. Determine the value of $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\frac{.10}{.3} = .333 \quad \boxed{C}$$

3. Determine the value of $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$\frac{.10}{.6} = .167 \quad \boxed{B}$$

Answer choices for Questions 1, 2, 3:

a. 0.1

b. 0.167

c. 0.333

d. 0.375

e. 0.75

$$.6 \cdot .3 = .10 \rightarrow .18 \neq .10$$

$$.6 \neq .333$$

$$.3 \neq .167$$

4. Which of the following cannot be used to test for independence?

a. $P(A) \cdot P(B) = P(A \cap B)$

b. $P(A) + P(B) = P(A \cup B)$

c. $P(A) = P(A|B)$

d. $P(B) = P(B|A)$

5. Are events A and B independent?

a. Yes, they are independent

b. No, they are dependent

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$.8 = .6 + .3 - x$$

$$.8 = .9 - x$$

$$-.1 = -x \quad x = .10$$

$$P(D) = .30$$

$$P(C|D) = .40$$

$$P(C|D^c) = .60$$

Examples: $P(D^c) = 1 - .30 = .70$ $P(C^c|D) = 1 - .40 = .60$ $P(C^c|D^c) = 1 - .60 = .40$

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those “felony” students, 40% go on to college. Of the ones who do not face a disciplinary action, 60% go on to college.

a. What is the probability that a randomly selected student both faced a disciplinary action and went on to college?

$$P(C \cap D) = P(C|D) \cdot P(D) = .40 \cdot .30 = .12$$

b. What percent of the students from the high school go on to college?

$$P(C \cap D^c) = P(C|D^c) \cdot P(D^c) = .60 \cdot .70 = .42$$
$$P(C) = P(C \cap D) + P(C \cap D^c) = .12 + .42 = .54$$

c. Show if events {faced disciplinary action} and {went to college} are independent or not.

$$P(C) = P(C|D) \rightarrow .54 \neq .40 \text{ Not independent}$$