

MATH 3307

Test Review 1

10 multiple choice questions, worth 60 points.

4 free response questions, worth 40 points.

Terms and Vocabulary;

Sample vs. Population

Discrete vs. Continuous

Standard Deviation vs. Variance

Combination vs. Permutation

Mean vs. Median vs. Mode

Reactivity to Outliers

A survey is conducted to determine the number of speeding tickets that an average teenager receives. To do this, DPS records are pulled for a random group of 150 teenagers.

Would this data be *(a) quantitative discrete*, (b) quantitative continuous, or (c) categorical?

Identify the sample and population for this study.

Sample: 150 randomly selected teenagers to have their DPS records pulled.

Population: All teenage drivers

Give an example of the two types of data (quantitative discrete, quantitative continuous, or categorical) that did not pertain to the previous question.

Quantitative Continuous: Weight, Height, Commuting distance, Time spent (fill-in-the-blank), etc.

Categorical: Hair color, eye color, blood type, favorite ice cream flavor, etc.

Twenty students were asked to provide their exam * grade for a recent test. The results were as follows: 78, 86, 80, 90, 95, 87, 86, 76, 77, 99, 100, 13, 85, 77, 86, 86, 97, 93, 81, 83

a. Give the mean, median, mode

```
> assign("x",c(78,86,80,90,95,87,86,76,77,99,100,13,85,77,86,86,97,93,81,83))
> mean(x)
[1] 82.75
> median(x)
[1] 86
```

b. Give the lower and upper quartile

```
> sort(x)
[1] 13 76 77 77 78 80 81 83 85 86 86 86 86 87 90 93 95 97 99 100
```

```
fivenum(x)
[1] 13.0 79.0 86.0 91.5 100.0 Lower Quartile: 79, Upper Quartile: 91.5
```

c. Give the range and interquartile range

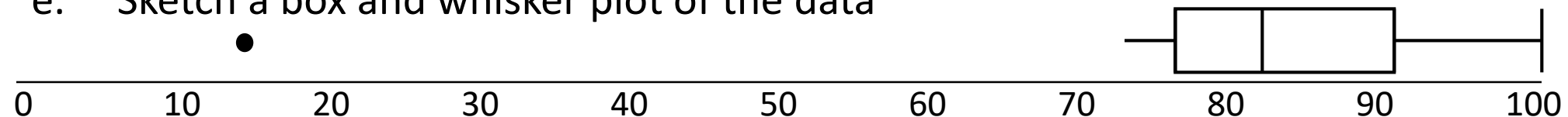
Range: $\text{Max} - \text{Min} = 100 - 13 = 87$; IQR: $Q3 - Q1 = 91.5 - 79 = 12.5$

d. Determine the interval for outliers

Low Outliers: $Q1 - 1.5 \cdot \text{IQR} = 79 - 1.5 \cdot 12.5 = 60.25$ (Values less than 60.25 are outliers; 13 is an outlier)

High Outliers: $Q3 + 1.5 \cdot \text{IQR} = 91.5 + 1.5 \cdot 12.5 = 110.25$ (Values greater than 110.25 are outliers; No high outliers)

e. Sketch a box and whisker plot of the data



A group of people contains 11 men and 8 women. You are going to select a committee of 6 to represent this group.

a. How many total committees are possible?

`choose(19, 6)`
`1) 27132`

b. How many committees are possible containing 4 men and 2 women?

`choose(11, 4) * choose(8, 2)`
`1) 9240`

c. What is the probability that a randomly selected committee will contain 4 men and 2 women?

$$\frac{\text{Part b Answer}}{\text{Part a Answer}} = \frac{9240}{27132} = 0.3406$$

<u>Population</u>
Men: 11
Women: 8
<hr/>
Total: 19

<u>Sample</u>
Men: 4
Women: 2
<hr/>
Total: 6

Adding an outlier to a group of existing data would have a significant effect on which of the following? (There are several correct answers.)

- Mean
- Median
- Mode
- Range
- Lower Quartile
- Upper Quartile
- Interquartile Range
- Variance
- Standard Deviation

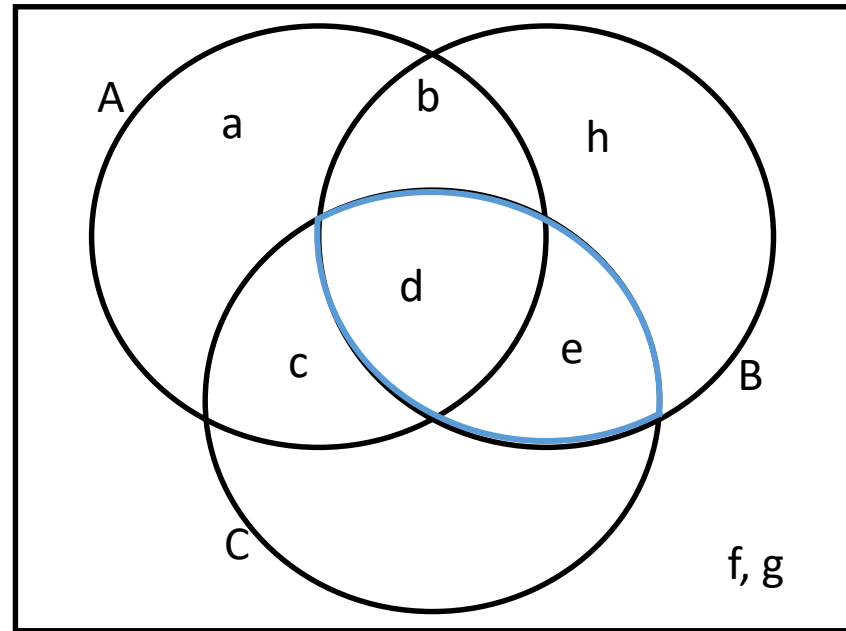
For the following:

$$U = \{a, b, c, d, e, f, g, h\}$$

$$A = \{a, b, c, d\}$$

$$B = \{b, d, e, h\}$$

$$C = \{c, d, e\}$$



- a. Find $B \cap C = \{d, e\}$ *This would be elements that are in both B and C*
- b. Find $A^c \cup B = \{b, d, e, f, g, h\}$ *This would be elements in A^c (below) or B*
 $A^c = \{e, f, g, h\}$ *This would be elements Not in A.*
- c. Find $(A \cap C^c)^c = \{c, d, e, f, g, h\}$ *Elements not in $A \cap C^c$ (below).*
 $C^c = \{a, b, f, g, h\}$ *Not in C. $A \cap C^c = \{a, b\}$ Elements in A and Not in C*
- d. Draw a Venn Diagram of the information *See Above*

In a graduating class, 85% of students had been employed during college, 65% had done volunteer work, and 55% had done both. What percent of the graduating class has either worked, done volunteer or both? What percent had done neither?

$$P(E \cup V) = P(E) + P(V) - P(E \cap V) = 0.85 + 0.65 - 0.55 = 0.95 \rightarrow 95\%$$

$$P(\text{Neither}) = P((E \cup V)^c) = 1 - P(E \cup V) = 1 - 0.95 = 0.05 \rightarrow 5\%$$

$$P(A) = 0.45, P(B) = 0.17, P(A \cup B) = 0.57$$

*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

a. Find $P(A \cap B)$

$$0.57 = 0.45 + 0.17 - x$$

$$0.57 = 0.62 - x$$

$$-0.05 = -x$$

$$0.05 = x$$

$$P(A \cap B) = 0.05$$

b. Find $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.17} = 0.2941$$

c. Find $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.45} = 0.1111$$

d. Are A and B independent events? **No** (Either of three explanations below is acceptable.)

$$P(A|B) \neq P(A)$$

$$P(B|A) \neq P(B)$$

$$P(A) \cdot P(B) \neq P(A \cap B)$$

*

The Department of Public Safety has put out information that the probability of having a certain number of “total loss” accidents in a driver’s lifetime is given by the following table:

X	0	1	2	3	4
P(X)	0.35	0.25	0.2	0.15	k

a. Find $P(X = 4)$

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$$

$$0.35 + 0.25 + 0.2 + 0.15 + k = 1$$

$$0.95 + k = 1 \rightarrow k = 0.05$$

b. Find the mean and standard deviation of number of accidents.

```
> assign("x",c(0,1,2,3,4))
> assign("p",c(.35,.25,.2,.15,.05))
> sum(x*p)
[1] 1.3
> sum(x^2*p)-sum(x*p)^2
[1] 1.51
> sqrt(1.51)
```

Mean: [1] 1.3

Standard Deviation: [1] 1.228821

c. Find $P(1 < X \leq 3)$

$$P(X = 2) + P(X = 3) = 0.2 + 0.15 = 0.35$$

d. Revising this data for total accidents (rather than total loss) creates a new Random Variable Y (created from increasing the X-value by 1 and then multiplying that result by 4). Find $E[Y]$ and $\sigma[Y]$.

$$Y=4(X+1)$$

$$Y=4X+4$$

$$E[Y] = 4 \cdot E[X] + 4 = 4(1.3) + 4 = 9.2$$

$$\sigma[Y] = 4 \cdot \sigma[X] = 4(1.2288) = 4.9152$$

A restaurant itemized its customers food preference with their beverage order during a weekend dinner shift (356 diners) with the following results:

	Burger	Pizza	Nachos	Sandwich	Salad
Iced Tea	15	28	11	35	21
Soda	25	32	12	28	10
Diet Soda	19	10	17	16	18
Beer	23	13	10	9	4

What is probability that someone ordering pizza will also order a beer?

$$P(B|P) = \frac{n(B \cap P)}{n(P)} = \frac{13}{28 + 32 + 10 + 13} = 0.1566$$

If a diner is drinking iced tea, what is the probability that he will order nachos?

$$P(N|T) = \frac{n(N \cap T)}{n(T)} = \frac{11}{15 + 28 + 11 + 35 + 21} = 0.1$$

A security consultant is interested in how many zeros are likely to show up in a randomly generated, 4-digit, ATM PIN.

Is this a binomial distribution? Why?

Yes. $n = 4$, $p = 0.1$. Trials are independent. Two possible outcomes (zero or not zero). We are interested in how many successes occur in a certain number of trials.

Determine the probability that all digits in the PIN will be zero. $x = 4$

```
dbinom(4,4,0.1)
[1] 1e-04      0.0001
```

Determine the probability that at least one digit will be zero. $x = 1, 2, 3, 4$

```
1-pbinom(0,4,0.1)
[1] 0.3439      1 - <"throw away">
```

Construct the probability distribution table for this situation.

X	0	1	2	3	4
P	0.656	0.292	0.048	0.0036	0.0001

Calculate the mean and standard deviation of this distribution.

$$\mu = n \cdot p = 4 \cdot 0.1 = 0.4 \quad \sigma = \sqrt{n \cdot p \cdot (1 - p)} = \sqrt{4 \cdot 0.1 \cdot (1 - 0.1)} = 0.6$$

```
· dbinom(0,4,0.1)
[1] 0.6561
· dbinom(1,4,0.1)
[1] 0.2916
· dbinom(2,4,0.1)
[1] 0.0486
· dbinom(3,4,0.1)
[1] 0.0036
· dbinom(4,4,0.1)
[1] 1e-04
· |
```

A casting director for a movie needs one blonde actress to play a certain role. Knowing that 20% of the population is blonde, what is the probability that he will need to audition no more than 10 actresses to find the first blonde?

(Before you answer, what kind of distribution is this?)

This is geometric distribution. Trials are independent, $p = 0.20$, n is unknown, two outcomes.

$X = 0, 1, 2, \dots, 10$ ("no more than 10" means less than or equal to 10)

```
pgeom(10-1, 0.20)  
[1] 0.8926258
```

Note: In RStudio, the first entry is $x-1$, not just x .