MATH 3307

Test 3 Review

10 Multiple Choice Questions:

(7 points each)

4 Written Response Questions

(3 points each)

Topics to know:

When to use each type of test.

z-test vs. t-test for means

Concluding based on p vs α

Effects of changes to the Confidence Interval

Hypothesis tests:

Test	Null Hypothesis	Test Statistic
One-sample z-test for means	$\mu = \mu_o$	$z = \dfrac{\overline{x} - \mu_o}{\dfrac{\sigma}{\sqrt{n}}}$
One-sample t-test for means	$\mu = \mu_o$	$t = \dfrac{\overline{x} - \mu_o}{\dfrac{s}{\sqrt{n}}}$; df = n–1
Matched Pairs t-test	$\mu_D = \mu_{D_0}$	$t = \dfrac{\overline{x}_D - \mu_D}{s / \sqrt{n}}$; df = n – 1
One-sample z-test for proportions	$p = p_o$	$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$
Two-sample t-test for means	$\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$	$t = \dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$; df=min(n1,n2)-1
Two-sample z-test for proportion	$p_1 - p_2 = 0$ or $p_1 = p_2$	$z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\left(\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}\right)}}$
χ^2 Goodness of fit test	no change	$\chi^2 = \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Confidence Intervals

General formula: *statistic* ± *margin of error*

One-sample z-test: $\overline{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$

Two-proportion z-test: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

One-sample t-test: $\overline{x} \pm t^* \dfrac{s}{\sqrt{n}}$

One-proportion z-test: $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Two-sample z-test: $(\overline{x}_1 - \overline{x}_2) \pm z^* \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Two-sample t-test: $(\overline{x}_1 - \overline{x}_2) \pm t^* \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

An 80% confidence interval was calculated to show the difference in passing percentages of the Biology Final Exam and Algebra Final Exam. The resulting interval was [.04, .12].

Answer the following, if it is possible to calculate:

- The difference in the sample proportions. (center interval) $\frac{.12+.04}{2} = \frac{.16}{2} = \boxed{.08}$
- The pass rate of the Algebra Final Exam. Cannot Calculate
- The margin of error for the difference between these passrates. $\frac{.12-.04}{2} = \frac{.08}{2} = \boxed{.04}$
- Are the two samples of equal size? Cannot Determine
- Was a t-test or a z-test used?
- Which final exam had the higher passrate?

Cannot Determine

$.12 - .08 = \boxed{.04}$

$.08 - .04 = \boxed{.04}$

A confidence interval is calculated using a two-sample t-test for means and the resulting interval was [10, 30]. Determine the difference in sample means and the margin of error of this interval.

Difference of means: (center value)

$$\frac{10+30}{2} = \frac{40}{2} = \boxed{20}$$

M of E:

$$\frac{30-10}{2} = \frac{20}{2} = \boxed{10}$$

$30 - 20 = 10$

$20 - 10 = 10$

A study was conducted to determine the mean number of traffic tickets a person receives by the age of thirty. A 90% confidence interval was calculated, yielding [4, 10] as the result. Give an explanation of this interval.

We are 90% certain that the mean number of traffic tickets a person receives by the age of 30 is between 4 and 10.

A SRS of 81 observations produced a mean of 250 with a standard deviation of 22. Determine the 95% confidence interval for the population mean.

$n = 81$

$\bar{x} = 250$

$s = 22$

CL: 95%

```
> 250-qt(1.95/2,80)*22/sqrt(81)
[1] 245.1354
> 250+qt(1.95/2,80)*22/sqrt(81)
[1] 254.8646
```

$[245.13, 254.87]$

A random sample of 64 observations produced a sample proportion of 0.23. Determine the 95% confidence interval for the population proportion.

$\hat{p} = 0.23$

$n = 64$

CL: 95%

```
 0.23-qnorm(1.95/2)*sqrt(0.23*(1-0.23)/64)
1] 0.1268979
 0.23+qnorm(1.95/2)*sqrt(0.23*(1-0.23)/64)
1] 0.3331021
```

$[0.127, 0.333]$

After performing a p-test, the P-Value is found to be smaller than the significance level (α). How should you proceed?

$P < \alpha$

RH_0

a. Reject the Null Hypothesis
b. ~~Accept the Alternate Hypothesis~~
c. ~~Fail to Reject the Null Hypothesis~~
d. ~~Accept the Null Hypothesis~~
e. ~~Fail to Reject the Alternate Hypothesis~~
f. ~~Perform a q-test to confirm results~~
g. ~~Accept them both, the more the merrier!~~
h. ~~Throw a party~~
i. ~~Any or All of the above are acceptable (*no one really hypothesis tests anyway*)~~

Which of the following will increase the width of confidence interval for the sample mean?

- Decrease the sample size
- Increase the confidence level
- Increase the sample size
- Decrease the confidence level
- Increase the variance
- Decrease the standard deviation
- Increase the value of the mean
- Decrease the value of the mean

The weights of pennies produced by the US Mint is determined to have a standard deviation of 0.2 grams. You wish to create a mean confidence interval of level 90%. How large of a sample of pennies should you select to have a margin of error of .02?

$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

$n = \left(\frac{z^* \sigma}{ME}\right)^2$

$\frac{ME}{1} = z^* \frac{\sigma}{\sqrt{n}} = \frac{z^* \sigma}{\sqrt{n}}$

```
(qnorm(1.90/2)*0.2/0.02)^2
] 270.5543
```

$\sqrt{n}\, ME = z^* \sigma$

$\sqrt{n} = \frac{z^* \sigma}{ME}$

$n = 271$

A one-sample z-statistic for a test of Ho: $\mu = 53$, and Ha: $\mu > 53$ based of 75 observations and calculations show $z = 1.837$.
Determine the p-value.

```
1-pnorm(1.837)
[1] 0.03310495
```

or 3.3%

An assortment of candies claims that their sample bag contains the following: 15% Snickers, 35% Milky Way, 25% Three Musketeers, 15% Almond Joy and 10% Mounds.
From a bag of 200 candies, you find there are 30 Snickers, 35 Milky Way, 40 Three Musketeers, 45 Almond Joy and 50 Mounds.
Use a $\chi 2$ test for goodness of fit to determine if the company's claim is accurate. (Use $\alpha = 0.01$)

Determine the Null and Alternate Hypothesis.

H_0: Expected Values are the Same as observed.

H_a: Expected Values are different than observed.

Candy Name	Expected Percents	Observed Amounts
Snickers	15%	30
Milky Way	35%	35
Three Musketeers	25%	40
Almond Joy	15%	45
Mounds	10%	50
	Total	200

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From a bag of 200 candies, you find there are 30 Snickers, 35 Milky Way, 40 Three Musketeers, 45 Almond Joy and 50 Mounds.
Use a $\chi 2$ test for goodness of fit to determine if the company's claim is accurate. (Use α = 0.01)

(Always shade the right side)

Find and sketch the rejection region.

```
qchisq(1-0.01,4)
] 13.2767
```

Candy Name	Expected Percents	Observed Amounts
Snickers	15%	30
Milky Way	35%	35
Three Musketeers	25%	40
Almond Joy	15%	45
Mounds	10%	50
	Total	200

Rej Reg:
$\chi^2 > 13.277$

An assortment of candies claims that their sample bag contains the following: 15% Snickers, 35% Milky Way, 25% Three Musketeers, 15% Almond Joy and 10% Mounds.
From a bag of 200 candies, you find there are 30 Snickers, 35 Milky Way, 40 Three Musketeers, 45 Almond Joy and 50 Mounds.
Use a $\chi 2$ test for goodness of fit to determine if the company's claim is accurate. (Use $\alpha = 0.01$)

Determine the value of the test-statistic

Candy Name	Expected Percents	Observed Amounts
Snickers	15%	30
Milky Way	35%	35
Three Musketeers	25%	40
Almond Joy	15%	45
Mounds	10%	50
	Total	200

```
> assign("expper",c(.15,.35,.25,.15,.10))
> assign("obs",c(30,35,40,45,50))
> sum(obs)
[1] 200
> expcount=expper*200
> expcount
[1] 30 70 50 30 20
> sum((obs-expcount)^2/expcount)
[1] 72
```

$\chi^2 = 72$

An assortment of candies claims that their sample bag contains the following: 15% Snickers, 35% Milky Way, 25% Three Musketeers, 15% Almond Joy and 10% Mounds.

From a bag of 200 candies, you find there are 30 Snickers, 35 Milky Way, 40 Three Musketeers, 45 Almond Joy and 50 Mounds.

Use a $\chi 2$ test for goodness of fit to determine if the company's claim is accurate. (Use $\alpha = 0.01$)

Determine the p-value.

```
1-pchisq(72,4)
1] 8.548717e-15
```

$P = 8.55 \times 10^{-15}$

$P = 0.00000000000000855 \approx 0$

Candy Name	Expected Percents	Observed Amounts
Snickers	15%	30
Milky Way	35%	35
Three Musketeers	25%	40
Almond Joy	15%	45
Mounds	10%	50
	Total	200

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Use a $\chi 2$ test for goodness of fit to determine if the company's claim is accurate. (Use α = 0.01)

Draw a Conclusion

Candy Name	Expected Percents	Observed Amounts
Snickers	15%	30
Milky Way	35%	35
Three Musketeers	25%	40
Almond Joy	15%	45
Mounds	10%	50
	Total	200

Method 1:

$p < \alpha$

$(p \approx 0)$ $(\alpha = .01)$

Method 2:

α falls within Rej. Reg

$72 > 13.277$

RH_0

To determine the effectiveness of a summer reading program, a sample of 50 children were tested for their reading speed before and after the program. The mean difference of this sample was an increased reading speed of 1.4 minutes for one page of text (with a standard deviation of 0.03 minutes). Determine if the reading program was effective of the program with a significance of 2%.

matched pairs t-test

Determine the Null and Alternate Hypothesis.

$H_0: \mu_D = 0$

$H_a: \mu_D > 0$

To determine the effectiveness of a summer reading program, a sample of 50 children were tested for their reading speed before and after the program. The mean difference of this sample was an increased reading speed of 1.4 minutes for one page of text (with a standard deviation of 0.03 minutes). Determine if the reading program was effective of the program with a significance of 2%.

$\alpha = 0.02$

$df = 49$

Determine the Rejection Region.

```
> qt(1-.01,49)
[1] 2.404892
```

$t > 2.405$

To determine the effectiveness of a summer reading program, a sample of 50 children were tested for their reading speed before and after the program. The mean difference of this sample was an increased reading speed of 1.4 minutes for one page of text (with a standard deviation of 0.03 minutes). Determine if the reading program was effective of the program with a significance of 2%.

Calculate the Test Statistic.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$\bar{x} = 1.4$

$\mu = 0$

$s = .03$

$n = 50$

```
(1.4-0)/(0.03/sqrt(50))
1] 329.9832
```

$t = 329.98$

To determine the effectiveness of a summer reading program, a sample of 50 children were tested for their reading speed before and after the program. The mean difference of this sample was an increased reading speed of 1.4 minutes for one page of text (with a standard deviation of 0.03 minutes). Determine if the reading program was effective of the program with a significance of 2%.

Determine the p-value.

```
1-pt(320.98,49)
1] 0
```

$p = 0\%$

To determine the effectiveness of a summer reading program, a sample of 50 children were tested for their reading speed before and after the program. The mean difference of this sample was an increased reading speed of 1.4 minutes for one page of text (with a standard deviation of 0.03 minutes). Determine if the reading program was effective of the program with a significance of 2%.

Draw a conclusion.

t is in the R.R

R H_0

$p = 0 < \alpha = .02$

R H_0

Various studies state that, worldwide, 1% of persons suffer from Autism. You believe that this number should be higher. You take an SRS of 500 people and find that 8 people have Autism.

Determine your Null and Alternate Hypothesis.

$H_0: p = .01$

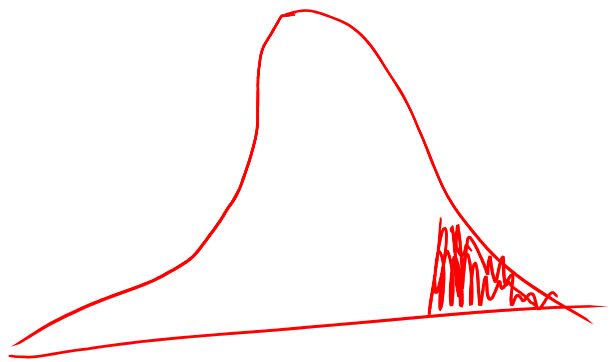
$H_A: p > .01$

Various studies state that, worldwide, 1% of persons suffer from Autism. You believe that this number should be higher. You take an SRS of 500 people and find that 8 people have Autism.

Determine your rejection region.

Right side.

Cannot determine value
(α not given)



Various studies state that, worldwide, 1% of persons suffer from Autism. You believe that this number should be higher. You take an SRS of 500 people and find that 8 people have Autism.

Calculate your test statistic.

$\hat{p} = \frac{8}{500}$

```
phat=8/500
(phat-.01)/sqrt(.01*(1-.01)/500)
1] 1.3484
```

$z = 1.3484$

Various studies state that, worldwide, 1% of persons suffer from Autism. You believe that this number should be higher. You take an SRS of 500 people and find that 8 people have Autism.

Determine your p-value.

(z-test)

proportions

```
1-pnorm(1.3484)
] 0.08876488
```

Various studies state that, worldwide, 1% of persons suffer from Autism. You believe that this number should be higher. You take an SRS of 500 people and find that 8 people have Autism.

Draw a conclusion. (No α given. Compare with 10% .. 0.10)

$p < .10$

($p = .08$)

R H_0

It is determined that the mean number of car accidents teenage drivers experience is 4. You collect data from 20 twenty-year olds and find their mean number of accidents was 6.3 with a standard deviation of 0.2. Test, with a significance of 5%, if there is doubt in the accepted mean.

Determine your Null and Alternate Hypothesis.

$H_0: \mu = 4$

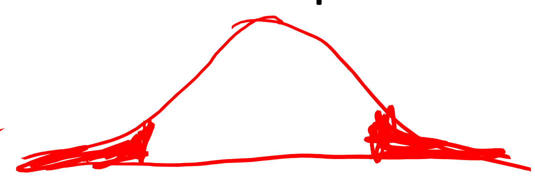
$H_a: \mu \neq 4$

It is determined that the mean number of car accidents teenage drivers experience is 4. You collect data from 20 twenty-year olds and find their mean number of accidents was 6.3 with a standard deviation of 0.2. Test, with a significance of 5%, if there is doubt in the accepted mean.

Determine your rejection region.

$H_a: \mu \neq 4$

sample stnd dev: t-test



$n = 20, \quad df = 19$

```
qt(0.05/2,19)
1] -2.093024
```

$t < -2.093$ or $t > 2.093$

It is determined that the mean number of car accidents teenage drivers experience is 4. You collect data from 20 twenty-year olds and find their mean number of accidents was 6.3 with a standard deviation of 0.2. Test, with a significance of 5%, if there is doubt in the accepted mean.

Calculate your test statistic. $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

```
 (6.3-4)/(0.2/sqrt(20))
1] 51.42956
```

$t = 51.43$

It is determined that the mean number of car accidents teenage drivers experience is 4. You collect data from 20 twenty-year olds and find their mean number of accidents was 6.3 with a standard deviation of 0.2. Test, with a significance of 5%, if there is doubt in the accepted mean.

Determine your p-value.

```
2*pt(-51.42956,19)
[1] 7.308731e-22
```

$P \approx 0$

It is determined that the mean number of car accidents teenage drivers experience is 4. You collect data from 20 twenty-year olds and find their mean number of accidents was 6.3 with a standard deviation of 0.2. Test, with a significance of 5%, if there is doubt in the accepted mean.

Draw a conclusion.

RR: $t < -2.093$ or $t > 2.093$

$t = 51$

Falls within R.R.

RH_0

Method 2:

$p = 0\%$ $\alpha = 5\%$

$p < \alpha$

RH_0

Explain why these changes would result in a confidence interval for a population mean decreasing in width (or margin of error).

- Decrease Confidence Level : This will mean our interval has less validity, so we can make it narrower (More specific but less certain)
- Decrease Standard Deviation This will show more uniformity in the sample, thereby making sample data stronger.
- Increase Sample Size This will bring the sample closer to the population → More accurate.

Popper 32:

Fill out choice E for Questions 1 – 5

Also, Popper 33, A for #1 – 10 in case you missed any.