Counting Techniques, Combinations, Permutations, Sets and Venn Diagrams

Sections 2.1 & 2.2

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Outline

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Beginning Example

In the city of Milford, applications for zoning changes go through a two-step process:

1. A review by the panning commission.
2. A final decision by the city council.

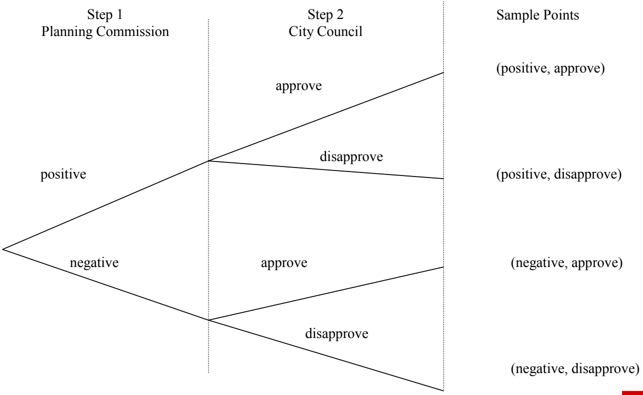
- At step 1 the planning commission reviews the zoning change request and makes a positive or negative recommendation concerning the change.
- At step 2 the city council reviews the planning commission's recommendation and then votes to approve or to disapprove the zoning change.

How many possible decisions can be made for a zoning change in Milford?

Counting Rules

- If an experiment can be described as a sequence of k steps with n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2)\ldots(n_k)$.
- A **tree diagram** can be used as a graphical representation in visualizing a multiple-step experiment.

Tree diagram



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Examples

- How many ways can you create a pizza choosing a meat and two veggies if you have 3 choices of meats and 4 choices for veggies?

- In how many ways can 6 people be seated in a row?

- How many possible outcomes can we have when rolling a pair of 6-sided die?

Permutations

It allows one to compute the number of outcomes when r objects are to be selected from a set of n objects where the order of selection is important. The number of permutations is given by

$$P_r^n = \frac{n!}{(n-r)!}$$

- Where $n! = n(n-1)(n-2)\cdots(2)(1)$
- Rocode for $n!$: factorial(n)

Allowing Repeated Values

When we allow repeated values, The number of orderings of n objects taken r at a time, with repetition is n^r.

- Example: In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

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Several Objects At Once

The number of permutations, P, of n objects taken n at a time with r objects alike, s of another kind alike, and t of another kind alike is $P = \frac{n!}{r!s!t!}$

- Example: How many different words (they do not have to be real words) can be formed from the letters in the word MISSISSIPPI?

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Objects Taken of Circular

The number of circular permutations of n objects is $(n-1)!$.

- Example: In how many ways can 12 people be seated around a circular table?

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Combinations

Counts the number of experimental outcomes when the experiment involves selecting r objects from a (usually larger) set of n objects. The number of combinations of n objects taken r unordered at a time is

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Rcode: choose(n,r)

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Examples

- In how many ways can a committee of 5 be chosen from a group of 12 people?

- In a manufacturing company they have to choose 5 out of 50 boxes to be sent to a store. How many ways can they choose the 5 boxes?

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Definitions

- A **set** is a collection of objects.
- The items that are in a set called **elements**.
- We typically denote a set by capital letters of the English alphabet.
- Examples: $A = \{knife, spoon, fork\}$, $B = \{2, 4, 6, 8\}$.
- The set B could also be written as
  $B = \{x|x \text{ are even whole numbers between 0 and 10}\}$.

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Notations of Sets

Notation	Description
$a \in A$	The object a is an element of the set A.
$A \subseteq B$	Set A is a subset of set B. That is every element in A is also in B.
$A \subset B$	Set A is a proper subset of set B. That is every element that is is in A is also in set B and there is at least one element in set B that is no in set A.
$A \cup B$	A set of all elements that are in A **or** B.
$A \cap B$	A set of all elements that are in A **and** B.
U	Called the **universal set**, all elements we are interested in.
A^C	The set of all elements that are in the universal set but are not in set A.

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Examples

The following are sets: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4, 5, 6, 9, 10\}$, $B = \{3, 4, 7, 8\}$, and $C = \{2, 3, 9, 10\}$

- $C \subset A$
- $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = U$
- $9 \in A$ but $9 \notin B$
- $A^C = \{7, 8\}$
- $A \cap B = \{3, 4\}$
- $A^C \cap C = \varnothing$ These means that the sets A^C and C are **disjoint**.
- $(B \cup C)^C = \{1, 5, 6\}$
- $A \cap B \cap C = \{3\}$

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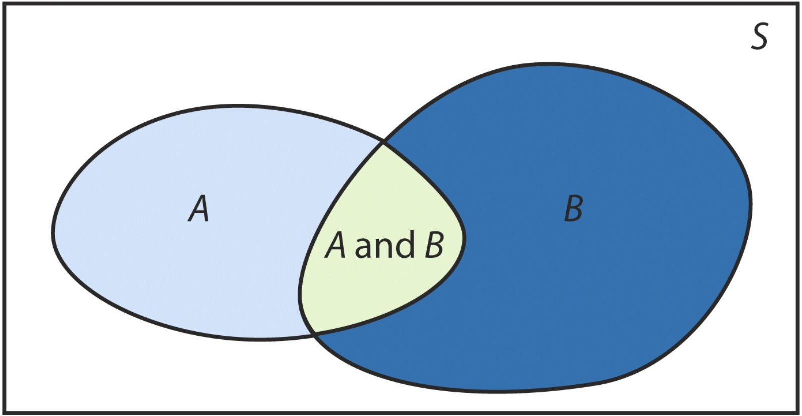
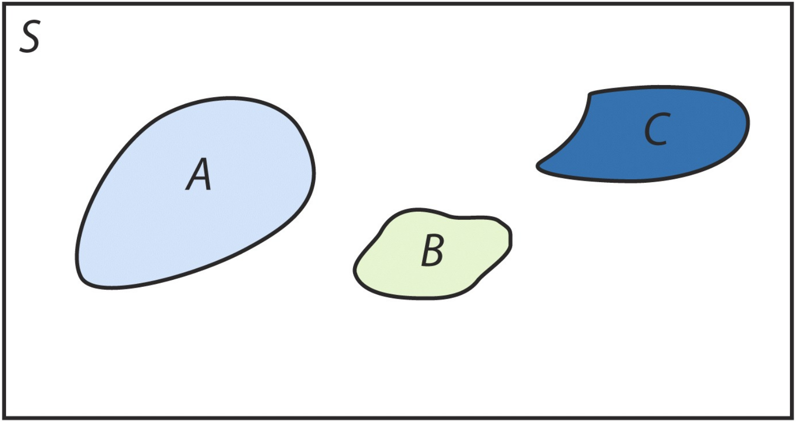
Definitions

- A **Venn diagram** is a very useful tool for showing the relationships between sets.
- Venn diagrams consist of a rectangle with one or more shapes (usually circles) inside the rectangle.
- The rectangle represents all of the elements that we are interested in for a given situation. This set is the universal set.

Graph of Venn Diagrams



Graph of Disjoint Events



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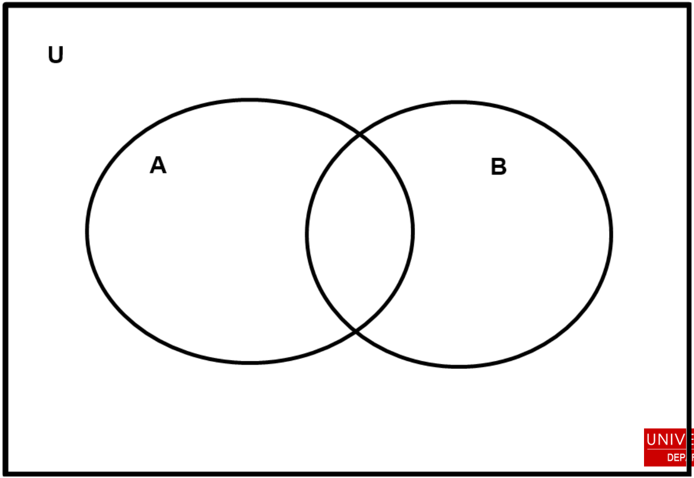
$A \cap B$



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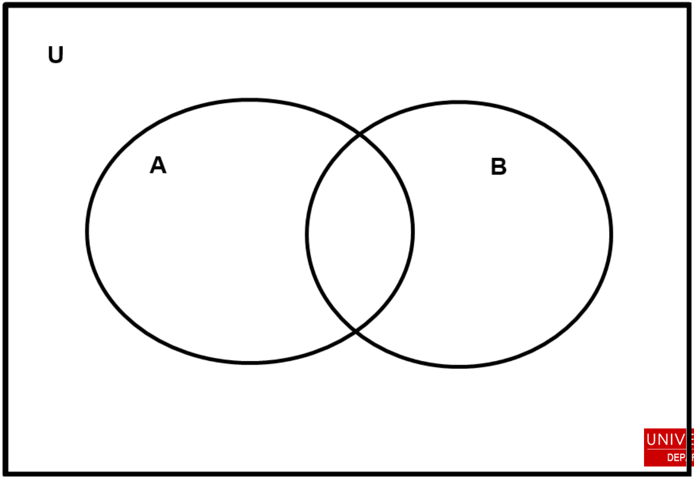
$A \cup B$



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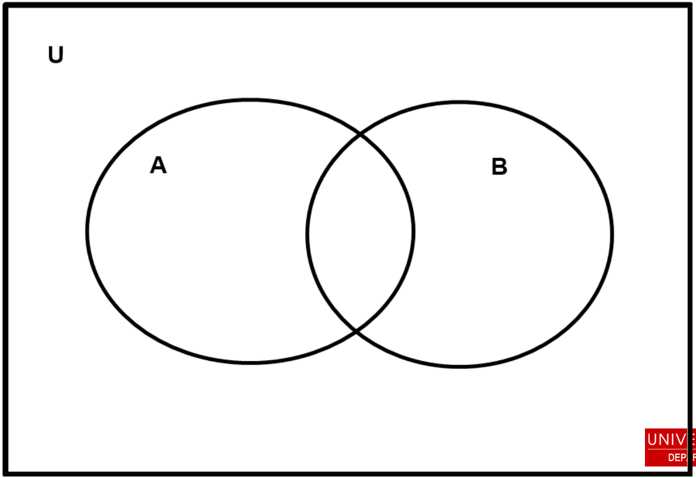
$A^C \cap B$



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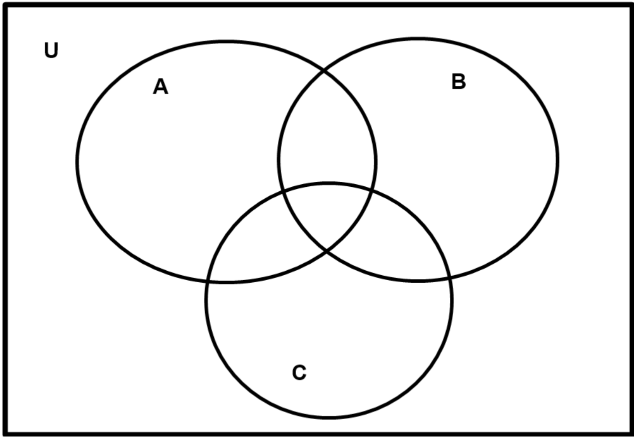
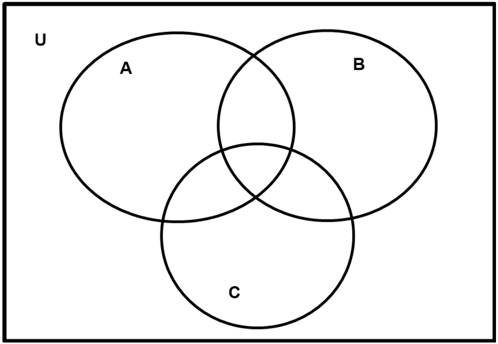
$A \cap B \cap C$



Soft Drink Preference

A group of 100 people are asked about their preference for soft drinks. The results are as follows: 55 like Coke, 25 like Diet Coke, 45 like Pepsi, 15 like Coke and Diet Coke, 5 like all 3 soft drinks, 25 like Coke and Pepsi, 5 only like Diet Coke (nothing else). Fill in the the Venn diagram with these numbers.



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