$Var(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \sum x^2 p_i$

$E(X) = \sum x p_i$

PRINTABLE VERSION

Binomial

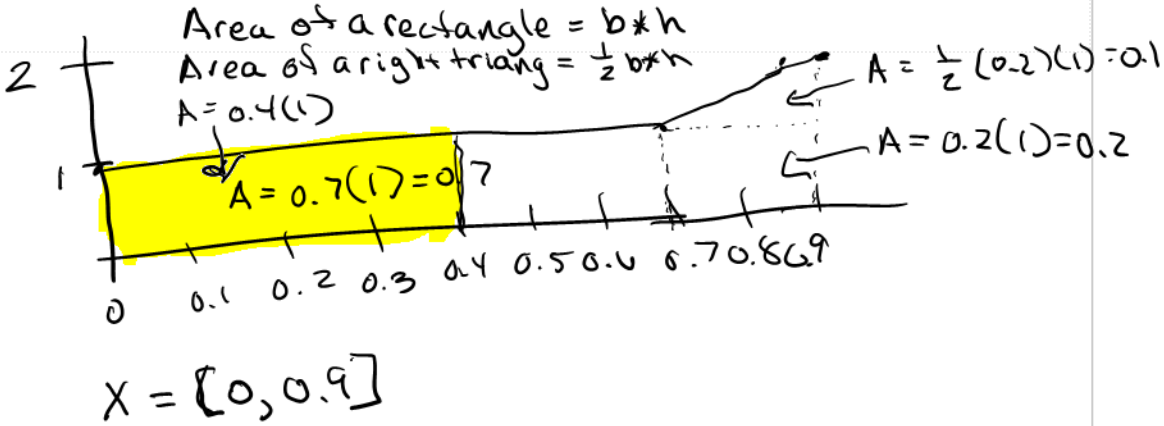
X = discrete $= 0, 1, 2, 3, \ldots, n$

Quiz 5

Question 1

Continuous Random Variables

Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (0.7, 1). The second goes from (0.7, 1) to (0.9, 2) in the xy-plane. What percent of observations fall below 0.40?



$X = [0, 0.9]$

a) 0.30

b) 0.20

c) 1.00

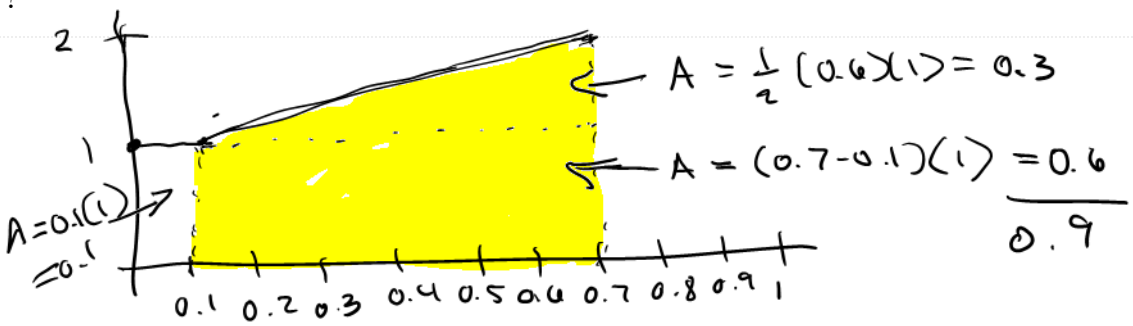
d) 0.40

e) 0.60

f) None of the above

Question 2

Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (0.1, 1). The second goes from (0.1, 1) to (0.7, 2) in the xy-plane. What percent of observations fall between 0.1 and 0.7?



a) 1.00

b) 0.90

c) 0.05

d) 0.10

e) 0.50

f) None of the above

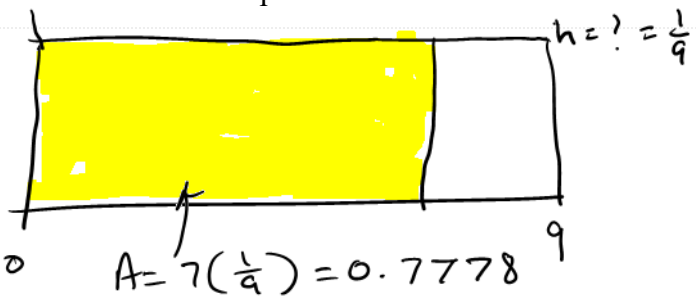
Question 3

Consider a uniform density curve defined from x = 0 to x = 9. What percent of observations fall below 7?

$A = bh$

$1 = 9h$

$h = 1/9$



a) 0.70

b) 0.11

c) 0.90

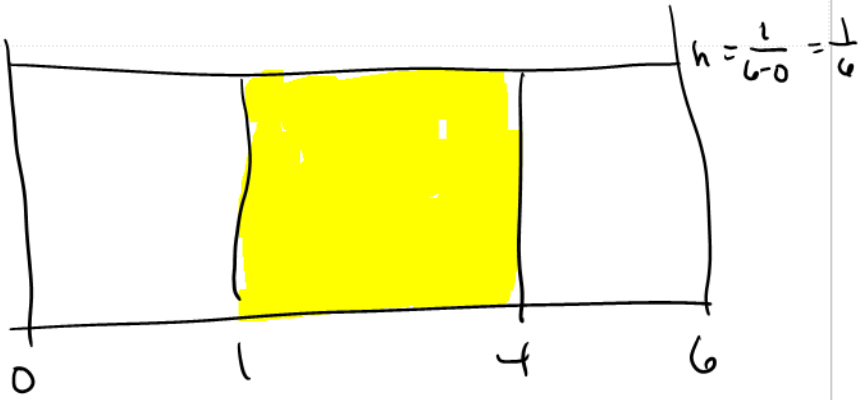
d) 0.78

e) 0.14

f) None of the above

Question 4

Consider a uniform density curve defined from x = 0 to x = 6. What percent of observations fall between 1 and 4?



a) 0.50

b) 0.17

c) 0.67

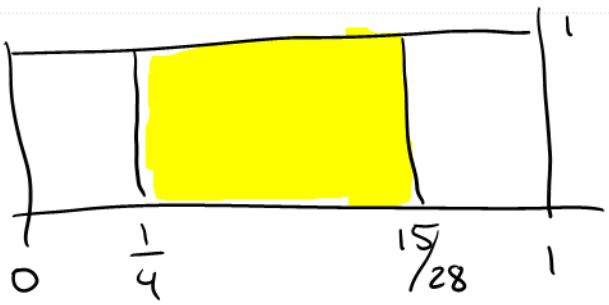
d) 0.25

e) 0.62

f) None of the above

Question 5

Consider a spinner that, after a spin, will point to a number between zero and 1 with "uniform probability". Determine the probability: $P(1/4 \le X \le 15/28)$.



$A = \left(\frac{15}{28} - \frac{1}{4}\right)(1)$

a) 0.29

b) 1.00

c) 0.54

d) 0.25

e) 0.71

f) None of the above

Question 6

The heights of students in a class are normally distributed with mean 68 inches and standard deviation 5 inches. Use the Empirical Rule to determine the interval that contains the middle 95% of the heights.

68-95-99.7 Rule

$\mu = 68$ (mean) $\sigma = 5$ (SD)

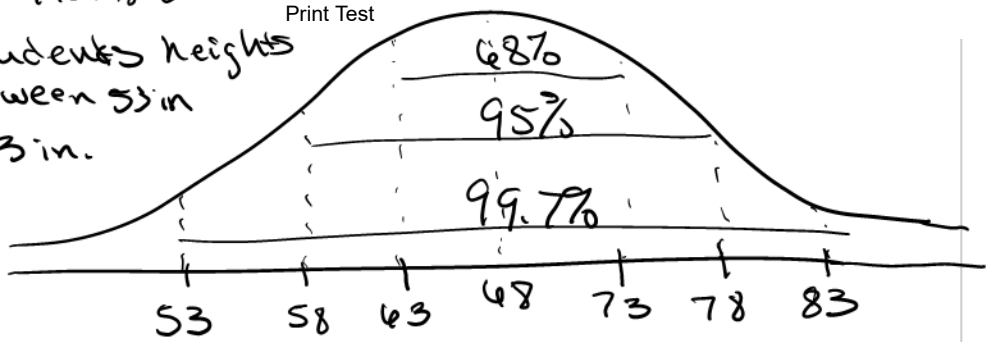
- About 68% are between 63 and 73 inches
- About 95% of the students height are between 58 & 78 in

a) [55, 81]

b) [58, 78]

- **c)** [53, 73]
- **d)** [53, 83]
- **e)** [63, 73]
- **f)** None of the above

-About 99.7% of the students heights are between 53 in and 83 in.

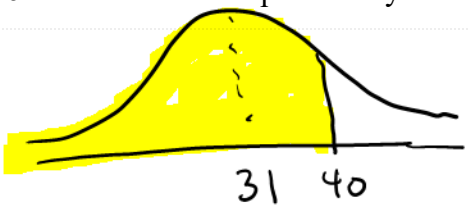


Question 7

$$z\text{-Score} = \text{Standard Score} = \frac{\text{value} - \text{mean}}{SD} = \frac{63-68}{5} = \frac{-5}{5} = -1$$

The length of time needed to complete a certain test is normally distributed with mean 31 minutes and standard deviation 6 minutes. Find the probability that it will take less than 40 minutes to complete the test.

- **a)** 0.5334
- **b)** 0.5000
- **c)** 0.0668
- **d)** 0.9332
- **e)** 0.4666
- **f)** None of the above



$$P(X < 40) = P\left(z < \frac{(40-31)}{6}\right) = P(z < 1.50) = 0.9332$$

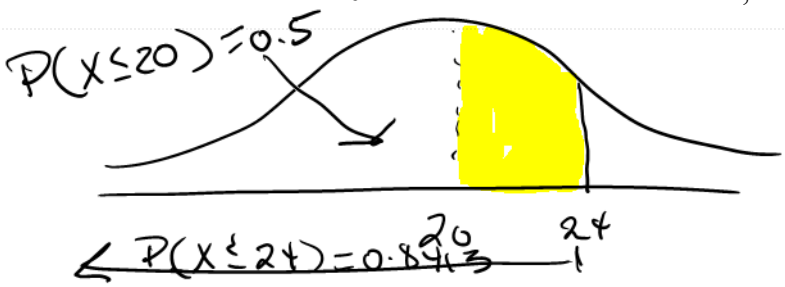
```
> pnorm(40,31,6)
[1] 0.9331928
```
← R

$$P(X \le x) = \text{pnorm}(x, \text{mean}, SD)$$

Question 8

If X is normally distributed with a mean of 20 and a standard deviation of 4, find $P(20 \le X \le 24)$.

- **a)** 0.641
- **b)** 0.341
- **c)** 0.841
- **d)** 0.441
- **e)** 0.541
- **f)** None of the above



$$P(20 \le X \le 24) = P(X \le 24) - P(X \le 20)$$

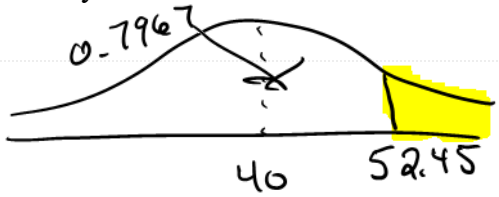
```
> pnorm(24,20,4)-pnorm(20,20,4)
[1] 0.3413447
```

$$P(20 \le X \le 24) = P\left(\frac{20-20}{4} \le z \le \frac{24-20}{4}\right) = P(0 \le z \le 1) = P(z \le 1) - P(z \le 0) = 0.8413 - 0.5 = 0.3413$$

Question 9

Suppose that X is normally distributed with a mean of 40 and a standard deviation of 15. What is $P(X \ge 52.45)$?

- **a)** 0.203
- **b)** 0.597



$$P(X \ge 52.45) = P\left(z \ge \frac{(52.45-40)}{15}\right) = P(z \ge 0.83) = 1 - 0.7967 = 0.2033$$

c) 0.206

d) 0.297

e) 0.207

f) None of the above

IAR: P(X≥52.45)

> 1-pnorm(52.45,40,15)
[1] 0.2032694

Question 10

Suppose that x is normally distributed with a mean of 50 and a standard deviation of 10. What is P($x \leq$ 74.50)?

a) 0.496

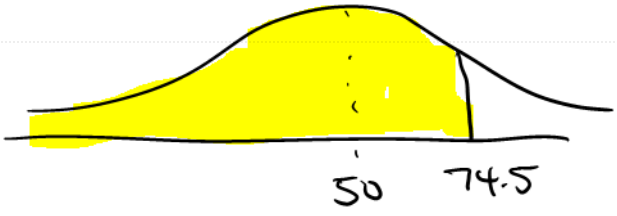
b) 0.493

c) 0.007

d) 0.993

e) 0.995

f) None of the above



P(X≤74.5) (x mean SD)

> pnorm(74.5,50,10)
[1] 0.9928572

Question 11

Suppose that x is normally distributed with a mean of 30 and a standard deviation of 3. What is P(27.42 $\leq x \leq$ 34.08)?

a) 0.718

b) 0.413

c) 0.309

d) 0.305

e) 0.415

f) None of the above

> pnorm(34.08,30,3)-pnorm(27.42,30,3)
[1] 0.7181905

Question 12

At a college the scores on the chemistry final exam are approximately normally distributed, with a mean of 76 and a standard deviation of 14. The scores on the calculus final are also approximately normally distributed, with a mean of 74 and a standard deviation of 15. A student scored 79 on the chemistry final and 81 on the calculus final. Relative to the students in each respective class, in which subject did the student do better?

Chem: mean = 76 SD = 14 x = 79

Calc: mean = 74 SD = 15 x = 81

a) Chemistry

b) Calculus

c) The student did equally well in each course

d) There is no basis for comparison

e) None of the above

Chem: $z = \frac{79-76}{14} = 0.2143$

Calc: $z = \frac{81-74}{15} = 0.467$

Question 13

Find a value of c so that $P(Z \leq c) = 0.48$.

a) 0.05

b) 0.95

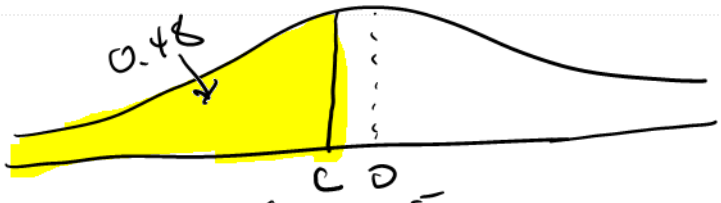
c) 0.45

d) -0.05

e) 1.05

f) None of the above

z mean(z) = 0
SD(z) = 1



c = -0.05

c = qnorm(0.48, 0, 1)

> qnorm(0.48,0,1)
[1] -0.05015358

$P(Z \leq -0.05) = pnorm(-0.05) = 0.48$

PRINTABLE VERSION

Quiz 6

Question 1

Suppose that x is normally distributed with a mean of 20 and a standard deviation of 15. What is $P(-7.45 \leq x \leq 54.95)$?

a) 0.956

b) 0.490

c) 0.468

d) 0.495

e) 0.466

f) None of the above

Question 2

Find a value of c so that $P(Z \geq c) = 0.71$.

a) -1.11

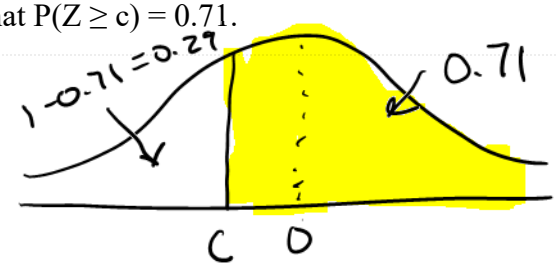
b) 0.75

c) -0.55

d) 0.55

e) 1.55

f) None of the above



C = qnorm(0.29)

> qnorm(0.29)
[1] -0.5533847

mean = 10
SD = 2
Find x such that
P(X ≥ x) = 0.71
X = qnorm(0.29,10,2)

> qnorm(0.29,10,2)
[1] 8.893231

Question 3

Suppose that x is normally distributed with a mean of 50 and a standard deviation of 15. What is $P(34.55 \leq x \leq 72.95)$?

a) 0.348

b) 0.785

c) 0.442

d) 0.353

e) 0.437

f) None of the above

Question 4

The length of time needed to complete a certain test is normally distributed with mean 31 minutes and standard deviation 6 minutes. Find the probability that it will take between 29 and 35 minutes to complete the test.

$P(29 \le X \le 35) = pnorm(35, 31, 6) - pnorm(29, 31, 6)$

a) 0.3694

b) 0.5000

c) 0.6219

d) 0.3781

e) 0.1890

f) None of the above

Question 5

The length of time needed to complete a certain test is normally distributed with mean 35 minutes and standard deviation 15 minutes. Find the probability that it will take more than 40 minutes to complete the test.

$P(x > 40) = 1 - pnorm(40, 35, 15)$

a) 0.3694

b) 0.6306

c) 0.5000

d) 0.1847

e) 0.8153

f) None of the above

Question 6

$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}}$ $E(\bar{x}) = \mu$ Normal if $n > 30$

Which of the following statements is <u>not</u> true?

a) The expected value of the sample mean, $\overline{X}$, is always the same as the expected value of X, the distribution of the population from which the sample was taken. ✓

b) The standard deviation of the sampling distribution $\overline{X}$ of sample mean = $\sigma/\sqrt{n}$ where σ is the standard deviation of X.

c) The larger the sample size, the better will be the normal approximation to the sampling distribution of sample mean.

d) The sampling distribution of the sample mean, $\overline{X}$, is always reasonably like the distribution of X, the distribution from which the sample is taken. As n gets large the distribution is more like the Normal distribution

e) The sampling distribution of sample mean is approximately normal, mound-shaped, and symmetric for $n > 30$ or $n = 30$.

f) None of the above

Question 7

Suppose a random sample of 60 measurements is selected from a population with a mean of 25 and a variance of 200. Select the pair that is the mean and standard error of $\bar{x}$. $\leftarrow SD(\bar{x})$

$n=60 \quad \mu = 25 \quad \sigma^2 = 200$

$\mu_{\bar{x}} = E(\bar{x}) = \mu = 25$

$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}} = \sqrt{\frac{200}{60}} = 1.825$

a) [25, 1.825]

b) [25, 1.925]

c) [60, 2.325]

d) [25, 2.225]

e) [25, 2.325]

f) None of the above

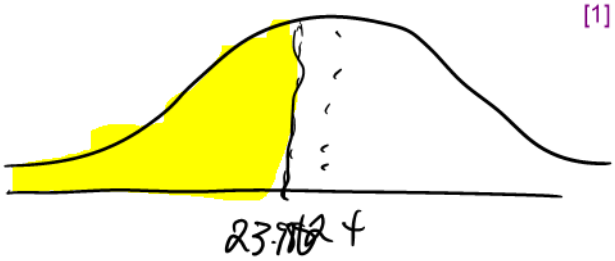
Question 8

A random sample of 400 24-ounce cans of fruit nectar is drawn from among all cans produced in a run. Prior experience has shown that the distribution of the contents has a mean of 24 ounces and a standard deviation of 0.24 ounce. What is the probability that the mean contents of the 400 sample cans is less than 23.988 ounces?

$n = 400 \quad \mu = 24 \quad \sigma = 0.24 \quad E(\bar{x}) = 24 \quad SD(\bar{x})$

$P(\bar{x} < 23.988) = pnorm(23.988, 24, \frac{0.24}{sqrt(400)})$

```
> pnorm(23.988,24,0.24/sqrt(400))
[1] 0.1586553
```



a) 0.199

b) 0.169

c) 0.209

d) 0.159

e) 0.179

f) None of the above

Question 9

The World Health Organization's (W.H.O.) recommended daily minimum of calories is 2600 per individual. The average number of calories ingested per capita per day for the US is approximately 2460 with a standard deviation of 500. If we take a random sample of 81 individuals from the US, what is the probability that the sample mean exceeds the W.H.O. minimum?

$\mu = 2460 \quad \sigma = 500 \quad n = 81 \quad E(\bar{x}) = 2460 \quad SD(\bar{x}) = \frac{500}{\sqrt{81}}$

$P(\bar{x} > 2600) =$

```
> 1-pnorm(2600,2460,500/sqrt(81))
[1] 0.005867742
```

a) 0.003

b) 0.002

c) 0.006

d) 0.009

e) 0.005

f) None of the above

Question 10

Current research indicates that the distribution of the life expectancies of a certain protozoan is normal with a mean of 46 days and a standard deviation of 10.5 days. Find the probability that a simple random sample of 49 protozoa will have a mean life expectancy of 47 or more days.

$\mu = 46 \quad \sigma = 10.5 \quad n = 49$

$P(\bar{x} \geq 47)$

```
> 1-pnorm(47,46,10.5/sqrt(49))
[1] 0.2524925
```

a) 0.7475

b) 0.5379

c) 0.4525

d) 0.2525

e) 0.5050

f) None of the above

Question 11

What effect does decreasing the sample size have on a distribution of sample means?

$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}}$

a) It will have more variation

b) It will not make any difference

c) It will have less variation

Question 12

Suppose that a random sample of size 36 is to be selected from a population with mean 50 and standard deviation 8. What is the approximate probability that $\bar{X}$ will be within 0.5 of the population mean?

$n = 36$ $\mu = 50$ $\sigma = 8$

$P(49.5 \le \bar{X} \le 50.5)$

```
> pnorm(50.5,50,8/sqrt(36))-pnorm(49.5,50,8/sqrt(36))
[1] 0.2923395
```

a) 0.5847

b) 0.0498

c) 0.4923

d) 0.7077

e) 0.2923

f) None of the above

Question 13

Suppose that a random sample of size 25 is to be selected from a population with mean 44 and standard deviation 7. What is the approximate probability that $\bar{X}$ will be more than 0.5 away from the population mean?

$P(\bar{X} \le 43.5 \text{ or } \bar{X} \ge 44.5)$

$= pnorm(43.5, 44, 7/\sqrt{25}) + 1 - pnorm(44.5, 44, 7/\sqrt{25})$

a) 0.2790

b) 0.0569

c) 0.4790

d) 0.7210

e) 0.5580

f) None of the above

Question 14

Lloyd's Cereal company packages cereal in 1 pound boxes (16 ounces). A sample of 64 boxes is selected at random from the production line every hour, and if the average weight is less than 15 ounces, the machine is adjusted to increase the amount of cereal dispensed. If the mean for 1 hour is 1 pound and the standard deviation is 0.2 pound, what is the probability that the amount dispensed per box will have to be increased?

$n = 64$ $\mu = 1$ lb $\sigma = 0.2$ lb

$P(\bar{X} < 15 \text{ oz}) = P(\bar{X} < \frac{15}{16} \text{ lb})$

$= pnorm(15/16, 1, 0.2/\sqrt{64})$

or $\mu = 16$ oz $\sigma = 0.2(16) = 3.2$

$= pnorm(15, 16, 3.2/\sqrt{64})$

a) 0.9938

b) 0.2062

c) 0.0124

d) 0.3773

e) 0.0062

f) None of the above

Question 15

In a large population, 67% of the households have cable tv. A simple random sample of 64 households is to be contacted and the sample proportion computed. What is the mean and standard deviation of the sampling distribution of the sample proportions?

$\hat{p}$ = sample percent

$E(\hat{p}) = p = 0.67$

$SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$

$= \sqrt{\frac{0.67(1-0.67)}{64}}$

$= 0.0588$

a) mean = 0.67, standard deviation = 1.7688

b) mean = 42.88, standard deviation = 0.0035

c) mean = 42.88, standard deviation = 0.0588

d) mean = 0.67, standard deviation = 0.0035

e) mean = 0.67, standard deviation = 0.0588

f) None of the above

Question 16

In a large population, 73% of the households have cable tv. A simple random sample of 100 households is to be contacted and the sample proportion computed. What is the probability that the sampling distribution of sample porportions is less than 70%?

$p = 0.73 \quad n = 100$ Normal dist

$P(\hat{p} < 0.7) = pnorm(0.7, 0.73, sqrt(0.73*(1-.73)/100$

a) 0.6373

b) 0.2496

c) 0.1248

d) 0.3627

e) 0.7504

f) None of the above