

Conditional Probability

PRINTABLE VERSION

$$P(A, \text{ given } B) = P(A | B) \text{ Quiz 3}$$

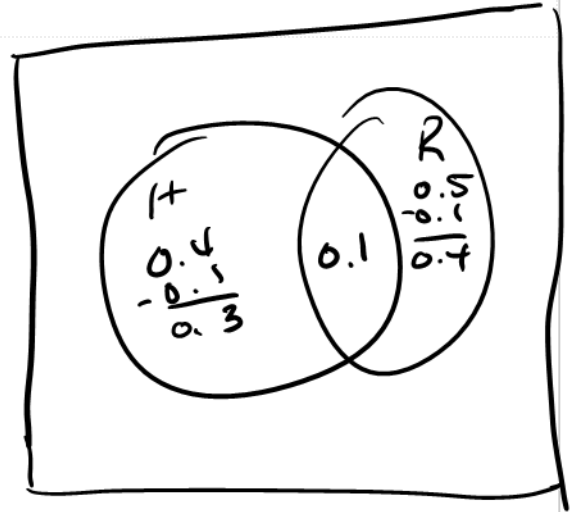
Question 1

$$= \frac{P(A \cap B)}{P(B)}$$

The probability that a randomly selected person has high blood pressure (the event H) is $P(H) = 0.4$ and the probability that a randomly selected person is a runner (the event R) is $P(R) = 0.5$. The probability that a randomly selected person has high blood pressure and is a runner is 0.1. Find the probability that a randomly selected person has high blood pressure, given that he is a runner.

- a) 0.25
- b) 1
- c) 0
- d) 0.20
- e) 0.11
- f) None of the above.

$$\begin{aligned} P(H, \text{ given } R) &= P(H | R) \\ &= \frac{P(H \cap R)}{P(R)} \\ &= \frac{0.1}{0.5} = 0.2 \end{aligned}$$

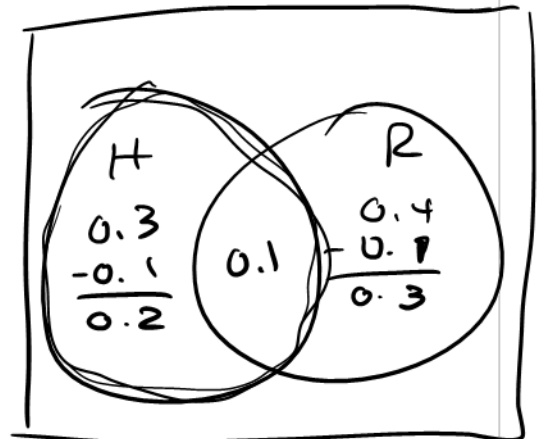


Question 2

The probability that a randomly selected person has high blood pressure (the event H) is $P(H) = 0.3$ and the probability that a randomly selected person is a runner (the event R) is $P(R) = 0.4$. The probability that a randomly selected person has high blood pressure and is a runner is 0.1. Find the probability that a randomly selected person is a runner, given that he has high blood pressure.

- a) 0.25
- b) 0
- c) 0.14
- d) 1
- e) 0.33
- f) None of the above.

$$\begin{aligned} P(R, \text{ given } H) &= P(R | H) \\ &= \frac{P(H \cap R)}{P(H)} \\ &= \frac{0.1}{0.3} = 0.3333 \end{aligned}$$



Question 3

The probability that a student correctly answers on the first try (the event A) is $P(A) = 0.4$. If the student answers incorrectly on the first try, the student is allowed a second try to correctly answer the question (the event B). The probability that the student answers correctly on the second try given that he answered incorrectly on the first try is 0.5. Find the probability that the student correctly answers the question on the first or second try.

$$P(A \cup B)$$

A = student answers correctly on the first try
 B = student answers correctly on the second try

a) 0.20

b) 0.90

c) 0.64

d) 0.70

e) 0.65

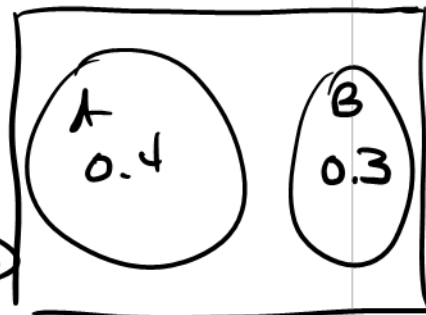
f) None of the above.

① $P(A \cap B) = 0$ disjoint events

$P(A) = 0.4$ $P(B, \text{ given } A^c) = 0.5$

② $P(A^c) = 1 - 0.4 = 0.6$
 $P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)}$

$0.5 = \frac{P(B \cap A^c) P(A^c)}{0.6} \Rightarrow P(B \cap A^c) = P(B) = 0.5(0.6)$



Question 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.3 - 0 = 0.7$

Given the following sampling distribution: $0.4 + 0.3 - 0 = 0.7$

X	-16	-14	-2	4	19
P(X)	1/50	1/20	3/50	1/20	

Discrete random variable

What is $P(X = 19)$?

a) 0.82

b) 0.81

c) 0.84

d) 0.19

e) 0.83

f) None of the above

$P(X = -16) = \frac{1}{50} = 0.02$

$P(X = -14) = \frac{1}{20} = 0.05$

$P(X = -2) = \frac{3}{50} = 0.06$

$P(X = 4) = \frac{1}{20} = 0.05$

$P(X = 19) = 1 - [0.02 + 0.05 + 0.06 + 0.05]$
 $= 1 - 0.18$
 $= 0.82$

Question 5

Given the following sampling distribution:

X	-20	-11	-4	11	17
P(X)	7/100	1/100	9/100	1/20	0.78

What is $P(X > -11)$?

a) 0.92

b) 0.95

c) 0.90

$P(X = 17) = 1 - [0.07 + 0.01 + 0.09 + 0.05]$
 $= 1 - 0.22$

$P(X \geq -11) = P(X = -11) + 0.92$

$P(X > -11) = P(X = -4 \text{ or } X = 11 \text{ or } X = 17)$
 $= P(X = -4) + P(X = 11) + P(X = 17)$
 $= 0.09 + 0.01 + 0.78 = 0.92$

- d) 0.93
- e) 0.91
- f) None of the above

$$\begin{aligned} \text{OR} \\ P(X > -11) &= 1 - [P(X = -20 \text{ or } X = -11)] \\ &= 1 - [0.07 + 0.01] \\ &= 1 - 0.08 \\ &= 0.92 \end{aligned}$$

Question 6

Given the following sampling distribution:

X	-20	-10	2	12	20
P(X)	$\frac{9}{100}$	$\frac{1}{25}$	$\frac{9}{100}$	$\frac{7}{100}$	

0.09 0.04 0.09 0.07 0.71

What is the mean of this sampling distribution?

- a) 0.2
- b) 12.9
- c) 13.2
- d) 13.0
- e) 0.8
- f) None of the above

Expected value $\text{mean}(X) = E(X)$

For means of discrete random variable

$$E(X) = \text{Sum}[X * P(X)]$$

$$\begin{aligned} &= -20(0.09) + (-10)(0.04) + 2(0.09) \\ &\quad + 12(0.07) + 20(0.71) \\ &= 13 \end{aligned}$$

R-Studio

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> x=c(-20,-10,2,12,20)
> px = c(.09,.04,.09,.07,.71)
> sum(x*px)
[1] 13.02
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Question 7

Suppose you have a distribution, X, with mean = 13 and standard deviation = 3. Define a new random variable Y = 4X - 5. Find the mean and standard deviation of Y.

- a) $E[Y] = 47; \sigma_Y = 7$
- b) $E[Y] = 47; \sigma_Y = 48$
- c) $E[Y] = 47; \sigma_Y = 12$
- d) $E[Y] = 52; \sigma_Y = 7$
- e) $E[Y] = 52; \sigma_Y = 48$
- f) None of the above

$$\text{mean}(X) = 13 \quad \text{SD}(X) = 3$$

$$Y = 4X - 5$$

$$\begin{aligned} \text{mean}(Y) &= \text{mean}(4X - 5) \\ &= 4 * \text{mean}(X) - 5 \\ &= 4(13) - 5 \\ &= 47 \end{aligned}$$

$$\begin{aligned} \text{SD}(Y) &= \text{SD}(4X - 5) = \text{SD}(4X) = 4 * \text{SD}(X) \\ &= 4(3) = 12 \end{aligned}$$

Question 8

In testing a certain kind of missile, target accuracy is measured by the average distance X (from the target) at which the missile explodes. The distance X is measured in miles and the sampling distribution of X is given

by:

X	0	10	50	100
P(X)	$\frac{1}{28}$	$\frac{1}{14}$	$\frac{1}{7}$	$\frac{3}{4}$

Calculate the mean of this sampling distribution.

a) 82.9b) 999.0c) 80.4d) 85.4e) 31.6f) None of the above

$$E(X) = 0\left(\frac{1}{28}\right) + 10\left(\frac{1}{14}\right) + 50\left(\frac{1}{7}\right) + 100\left(\frac{3}{4}\right)$$

$$= 0 \cdot \frac{1}{28} + 10 \cdot \frac{1}{14} + 50 \cdot \frac{1}{7} + 100 \cdot \frac{3}{4}$$

$$= [1] 82.85714$$

Question 9

In testing a certain kind of missile, target accuracy is measured by the average distance X (from the target) at which the missile explodes. The distance X is measured in miles and the sampling distribution of X is given by:

X	0	10	50	100
P(X)	$\frac{1}{34}$	$\frac{1}{17}$	$\frac{2}{17}$	$\frac{27}{34}$

Calculate the variance of this sampling distribution.

a) 29.4b) 288.5c) 4873.6d) 85.9e) 865.4f) None of the above

$$\text{Var}(X) = E(X^2) - E(X)^2$$

$$E(X^2) = 0^2\left(\frac{1}{34}\right) + 10^2\left(\frac{1}{17}\right) + 50^2\left(\frac{2}{17}\right) + 100^2\left(\frac{27}{34}\right)$$

$$= 8241.176$$

$$E(X) = 0\left(\frac{1}{34}\right) + 10\left(\frac{1}{17}\right) + 50\left(\frac{2}{17}\right) + 100\left(\frac{27}{34}\right)$$

$$= 85.88235$$

$$\text{Var}(X) = E(X^2) - E(X)^2$$

$$= 8241.176 - 85.88235^2$$

$$= 865.398$$

$$\text{SD}(X) = \sqrt{865.398}$$

Question 10

Suppose you want to play a carnival game that costs 6 dollars each time you play. If you win, you get \$100. The probability of winning is $\frac{1}{50}$. What is the expected value of the amount the carnival stands to gain?

a) 2.00

X = amount the carnival gain

b) 4.20c) 3.70d) 4.00e) -4.00f) None of the above

	X	P(X)
Player wins	-94	$\frac{1}{50} = 0.02$
Player loose	+6	$1 - 0.02 = 0.98$

$$E(X) = -94(0.02) + 6(0.98) = 4$$

Question 11

Suppose you want to play a carnival game that costs 9 dollars each time you play. If you win, you get \$100. The probability of winning is $\frac{1}{50}$. What is the expected value of the amount that you, the player, stand to gain?

X = amount the player gains

a) -7.30b) -7.00c) -6.80d) -2.00e) 7.00f) None of the above

	X	P(X)
Player w.r	91	$\frac{1}{50} = 0.02$
Player loose	-9	0.98

$$E(X) = 91(0.02) + (-9)(0.98) = -7$$

Question 12

A random sample of 2 measurements is taken from the following population of values: 1, 2, 4, 5, 8. What is the probability that the range of the sample is 7?

a) 0.2b) 0.4c) 0.5d) 0.3e) 0.1f) None of the above

$$P(\text{range} = 7) = \frac{n(\text{range} = 7)}{n(\text{choose 2 out of 5})}$$

$$= \frac{1}{C(5, 2)} = \frac{1}{10} = 0.1$$

range = highest - Lowest 1, 2 range = 1

4, 8 range = 4 4 range = 2 1, 4 range = 3

5, 5 range = 0 5 range = 3 1, 5 range = 4

5, 8 range = 3 2, 8 range = 6 1, 8 range = 7

Question 13

A furniture store is having a sale on sofas and you're going to buy one. The advertisers know that buyers get to the store and that 1 out of 7 buyers change to a more expensive sofa than the one in the sale advertisement.

Let X be the cost of the sofa. What is the average cost of a sofa if the advertised sofa is \$150 and the more expensive sofa is \$300?

a) 171.43

b) 168.75

c) 225.00

d) 278.57

e) 171.65

f) None of the above

$X = \text{cost of sofa}$

	x	$P(x)$
ad	150	$\frac{6}{7}$
expensive	300	$\frac{1}{7}$

$$E(x) = 150\left(\frac{6}{7}\right) + 300\left(\frac{1}{7}\right)$$

$$= 171.43$$