Probability Rules

1. Probability is between 0 and 1, inclusive. For any event, A,
   $0 \leq P(A) \leq 1$.

2. Total probability is 1. If we have a sample space S
   $P(S) = 1$.

3. Addition Rule: For any two events A and B
   $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   ("and")

4. Complent Rule: For any event A, with probability P(A)
   $P(A^c) = 1 - P(A)$

5. Multiplication Rule: For any two events, A and B,
   $P(A \text{ and } B) = P(A \cap B) = P(A) * P(B|A)$ or $P(B) * P(A|B)$
   where $P(A|B)$ and $P(B|A)$ are conditional probabilities.

If two events are mutually extusive, then $P(A \cap B) = 0$

If two events are independent, then, $P(A|B) = P(A)$

If independent $P(A \cap B) = P(B) * P(A|B)$
$= P(B) * P(A)$
$\neq 0$ Not mutually exclusive

Discrete Random Variables Probability Distribution

Example Quiz 3 Question 8

X	0	10	50	100
P(X)	$\frac{1}{40}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{33}{40}$
	0.025	0.05	0.1	0.825 = 1

mean $= E(X) = 0(0.025) + 10(0.05) + 50(0.1) + 100(0.825) = 88$

$\text{var}(X) = E(X^2) - [E(X)]^2 = 8505 - 88^2 = 761$ $SD(x) = \sqrt{761} = 27.58623$

$E(X^2) = 0^2(0.025) + 10^2(0.05) + 50^2(0.1) + 100^2(0.825) = 8505$

PRINTABLE VERSION

Quiz 4

Question 1

Suppose you have a distribution, X, with mean = 5 and standard deviation = 4. Define a new random variable Y = 8X - 5. Find the mean and standard deviation of Y.

mean = E(X) = 5 SD(X) = σ_X = 4

mean(Y) = E(Y) = E(8X - 5) = 8 E(X) - 5
= 8(5) - 5 = 35

σ_Y = SD(Y) = SD(8X - 5) = SD(8X) = 8 SD(X)
= 8(4) = 32

a) E[Y] = 35; σ_Y = 32

b) E[Y] = 40; σ_Y = 27

c) E[Y] = 35; σ_Y = 256

d) E[Y] = 40; σ_Y = 256

e) E[Y] = 35; σ_Y = 27

f) None of the above

Question 2

If a random variable, X, is binomial then:

Which statement is not true for a binomial distribution with n = 10 and p = 1/20 ? p = 0.05 q = 1 - 0.05 = 0.95

1. We have a fixed number of observations (random samples), n.
2. We have for each sample two outcomes, success or failure.
3. X = number of successes out of n trials = 0, 1, 2, 3, ..., n
4. Fixed probability of success for each trial (sample), p.
5. P(X = r) = C(n, r) $p^r (1-p)^{(n-r)}$

a) The highest probability occurs when **x** equals 0.5000 X ≠ 0.5

b) The number of trials is equal to 10 = n

c) The standard deviation is 0.6892

$\sigma_X = \sqrt{np(1-p)} = \sqrt{10(0.05)(1-0.05)} = 0.6892$

d) The probability that **x** equals 1 is 0.3151

P(X = 1) = C(10, 1) * $0.05^1 * 0.95^9$ = 0.3151

e) The mean equals 0.5000

E(X) = np = 10(0.05) = 0.5

f) None of the above

Question 3

In testing a new drug, researchers found that 3% of all patients using it will have a mild side effect. A random sample of 8 patients using the drug is selected. Find the probability that none will have this mild side effect.

p = 0.03 n = 8 binomial

q = 0.97

P(X = 0) = C(8, 0) * $0.03^0 * 0.97^8 = 0.97^8$ = 0.7837

> dbinom(0,8,0.03)
[1] 0.7837434

P(X = r) = dbinom(r, n, p)

a) 0.7837

b) 0.0300

c) 0.9700

d) 0.6791

e) 0.5305

f) None of the above

Question 4

In testing a new drug, researchers found that 3% of all patients using it will have a mild side effect. A random sample of 14 patients using the drug is selected. Find the probability that at least one will have this mild side effect.

$p = 0.03 \quad n = 14 \quad X = 0, 1, 2, \ldots, 14$ "greater than or equal to"

$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) + \cdots + P(X=14)$

$= 1 - P(X=0)$

$= 1 - \text{dbinom}(0, 14, 0.03)$

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> 1-dbinom(0,14,0.03)
[1] 0.3471637
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a) 0.5482

b) 0.3472

c) 0.4099

d) 0.0300

e) 0.9700

f) None of the above

Question 5

In testing a new drug, researchers found that 6% of all patients using it will have a mild side effect. A random sample of 11 patients using the drug is selected. Find the probability that exactly two will have this mild side effect.

$p = 0.06 \quad n = 11$ Binomial

$P(X=2) = \text{dbinom}(2, 11, 0.06)$

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> dbinom(2,11,.06)
[1] 0.113453
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a) 0.1635

b) 0.1435

c) 0.1535

d) 0.1135

e) 0.1235

f) None of the above

Question 6

A manufacturer of matches randomly and independently puts 22 matches in each box of matches produced. The company knows that one-tenth of 5 percent of the matches are flawed. What is the probability that a matchbox will have one or fewer matches with a flaw?

a) 0.09901

$n = 22 \quad p = 0.5\% = 0.005$

$P(X \leq 1) = P(X=0) + P(X=1)$ cumulative probability

= pbinom(1, 22, 0.005)

> pbinom(1,22,.005)
[1] 0.9945966

b) 0.8956

c) 0.9946

d) 0.9950

e) 0.004500

f) None of the above

Question 7

Suppose you have a binomial distribution with $n = 14$ and $p = 0.3$. Find $P(5 \le X \le 9)$.

$P(5 \le X \le 9) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9)$

> sum(dbinom(5:9,14,0.3))
[1] 0.4141332

$= P(X \le 9) - P(X < 5)$

$= P(X \le 9) - P(X \le 4)$

= pbinom(9, 14, 0.3) − pbinom(4, 14, 0.3)

> pbinom(9,14,0.3)-pbinom(4,14,0.3)
[1] 0.4141332

a) 0.4141

b) 0.6141

c) 0.5075

d) 0.4075

e) 0.4581

f) None of the above

Question 8

Each year a company selects a number of employees for a management training program. On average, 40 percent of those sent complete the program. Out of the 41 people sent, what is the probability that exactly 8 complete the program?

$p = 40\% = 0.4 \quad n = 41$

$P(X = 8) = dbinom(8, 41, 0.4) = 0.0029$

a) 0.0469

b) 0.0044

c) 0.1044

d) 0.2029

e) 0.0029

f) None of the above

Question 9

Each year a company selects a number of employees for a management training program. On average, 80 percent of those sent complete the program. Out of the 15 people sent, what is the probability that 11 or more complete the program?

$p = 80\% = 0.8 \quad n = 15$

a) 0.6481

b) 0.4518

c) 0.8357

d) 0.3518

e) 0.1642

f) None of the above

$P(X \geq 11) = 1 - P(X < 11) = 1 - P(X \leq 10)$

$= 1 - pbinom(10, 15, 0.8)$

$= 0.8357$

Question 10

A fish tank in a pet store has 21 fish in it. 9 are orange and 12 are white. Determine the probability that if we select 4 fish from the tank, at least 2 will be white.

a) 0.2114

b) 0.8105

c) 0.5714

d) 1.0305

e) 0.1894

f) None of the above

9 o, 12 w, 21

2 or 1 or 0 orange

2 or 3 or 4 w

4

$P(2w \text{ and } 2o) + P(3w \text{ and } 1o) + P(4w \text{ and } 0o)$

$\frac{choose(12,2) * choose(9,2)}{choose(21,4)} + \frac{choose(12,3) * choose(9,1)}{choose(21,4)} + \frac{choose(12,4) * choose(9,0)}{choose(21,4)}$

Question 11

Identify the following distribution as binomial, geometric or neither.
A quarter back completes 44% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass.

a) Binomial

b) Geometric

c) Neither

If X is geometric

X = # of failures = 0, 1, 2, ...

p = probability of success

$P(X = r) = p(1-p)^r$

Question 12

A quarter back completes 48% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass. What is the probability that the quarterback throws 4 incomplete passes before he has a completion?

a) 0.0402

b) 0.9649

$p = 0.48$ Geometric

$P(X = 4) = 0.48(1-0.48)^4 = 0.0350$

$= dgeom(4, 0.48)$

$P(X = r) = dgeom(r, p)$

c) 0.0350

d) 0.0182

e) 0.9817

f) None of the above

Question 13

A quarter back completes 22% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass. Determine the probability that it takes more than 11 attempts before he completes a pass.

$p = 0.22$

$P(X > 11) = (1 - 0.22)^{11} = 0.065$

$P(X > r) = (1 - p)^r$

a) 0.0363

b) 0.9349

c) 0.0650

d) 0.9856

e) 0.0143

f) None of the above

Question 14

Joe has an 37% probability of passing his statistics quiz 4 each time he takes it. What is the probability he will take no more than 5 tries to pass it?

$p = 0.37$ $P(X \leq 5) = 1 - P(X > 5)$

$= 1 - (1 - 0.37)^5$

$= 0.9007$

a) 0.0587

b) 0.9632

c) 0.0992

d) 0.0367

e) 0.9007

f) None of the above

Question 15

Joe has an 19% probability of passing his statistics quiz 4 each time he takes it. How many times should Joe expect to take his quiz before passing it?

$P = 0.19$

$E(x) = \frac{1}{p}$

$E(x) = \frac{1}{0.19} = 5.26$

a) 4

b) ○ 10

c) ○ 6

d) ○ 5

e) ○ 57

f) ○ None of the above