

# Math 2311

## TEST 1 REVIEW SHEET

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#1 – 20, Define the following:

1. Population data
2. Sample data
3. Categorical variables
4. Quantitative variables (know the difference between discrete and continuous)

5. Mean
6. Median
7. Mode
8. Range
9. Five number summary
10. IQR
11. Outliers
12. Standard deviation
13. Variance

- 14. Combination  $\rightarrow$  order does not matter  $nCr$
- 15. Permutation  $\rightarrow$  order does matter  $nPr$
- 16. Intersection  $\cap$  and - belongs to both sets
- 17. Union  $\cup$  or - either or both
- 18. Probability

- $n+p$  19. Binomial Distribution  $>$  know difference
- $p$  20. Geometric Distribution  $\nearrow$  looking for 1st success

21. What kind of variable? Categorical or quantitative? If quantitative, discrete or continuous?

- a. Score on the final exam (out of 100 points) as recorded on report card.

Quantitative, discrete

- b. Final grade for the course (A, B, C, D, F).

Categorical

- c. The amount a person grew (in height) in a year.

Quant. continuous

- d. The number of classes a student missed.

Quant, discrete

22. Twelve people were asked how many movies they saw last month. The results were:

2   6   1   3   4   2   1   5   3   6   4   5

- a. Find the mean and median.

< mean = 3.5  
median = 3.5  $\left(\frac{3+4}{2}\right)$

- b. Find the variance and the standard deviation.

Var =  $6^2 = 3.1818$   
Sd. dev = 6 = 1.7838

- c. Based on the values you get, what can you say about the shape of the distribution of the data set? Explain briefly.

Symmetrical (mean=median)

- d. Find the five-number summary.

min   Q1   med   Q3   max  
1   2   3.5   5   6

- e. Determine the interval for outliers.

$IQR = Q3 - Q1 = 3$

$1.5 \cdot IQR = 4.5$     $[Q1 - 4.5, Q3 + 4.5]$   
 $[-2.5, 9.5]$

23. Suppose that from a group of 9 men and 8 women, a committee of 5 people is to be chosen.

a. In how many ways can the committee be chosen?

$$17C_5 = 6188$$

b. In how many ways can the committee be chosen so that there are exactly 3 men and 2 women?

$$9C_3 \cdot 8C_2 = 2352$$

c. What is the probability that the committee has exactly 3 men and 2 women?

$$\frac{2352}{6188}$$

24. How many different words can be formed from the letters in the word STATISTICS? (they do not have to be real words)

.....  
3 S    3 T    2 I

$$\frac{10!}{3! \cdot 3! \cdot 2!}$$

$$50,400$$

25. Use the following information:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{3, 4, 6, 8\}$$

$$B = \{1, 2, 5, 9\}$$

$$C = \{2, 4, 6, 8, 10\}$$

a.  $A \cap C =$

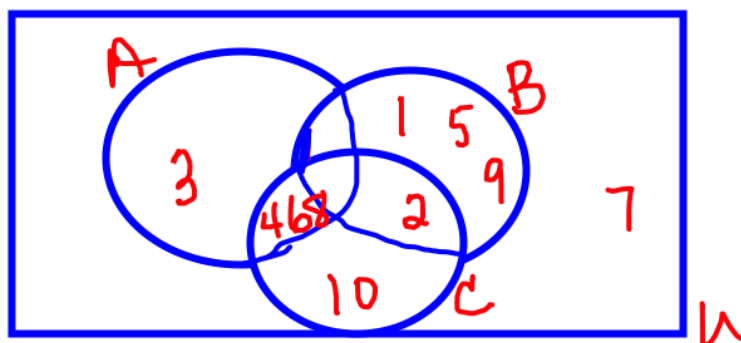
$$\{4, 6, 8\}$$

b.  $B^c \cup A =$

$$\{3, 4, 6, 7, 8, 10\}$$

c.  $(B \cup C)^c = B^c \cap C^c = \{3, 7\}$

d. Draw a Venn diagram for the information.



26. Suppose that 58% of all customers of a large insurance agency have automobile policies with the agency, 42% have homeowner's policies, and 23% have both. What is the probability that the customer has at least one of the policies?

$A = \text{auto}$   $H = \text{home}$

$$P(A) = .58$$

$$P(H) = .42$$

$$P(A \cap H) = .23$$

$$\begin{aligned} P(A \cup H) &= P(A) + P(H) - P(A \cap H) \\ &= .58 + .42 - .23 \\ &= .77 \end{aligned}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

27. Suppose  $P(A) = 0.72$ ,  $P(B) = 0.46$  and  $P(A \cup B) = 0.86$

a. Find  $P(A \cap B)$

$$= .32$$

$$.86 = .72 + .46 - x$$

$$.86 = 1.18 - x$$

$$x = .32$$

b. Find  $P(A|B)$

$$P(A|B) = P(A \cap B) / P(B) = .32 / .46 = .6957$$

c. Find  $P(B|A)$

$$P(B|A) = P(B \cap A) / P(A) = .32 / .72 = .4444$$

NO d. Area A and B independent?

$$P(A) \cdot P(B) \stackrel{?}{=} P(A \cap B) \quad P(B) \neq P(B|A)$$

$$(.46)(.72) \stackrel{?}{=} .32 \rightarrow .3312 \neq .32$$

28. Gabelli Partners is planning a major investment. The amount of profit X is uncertain but a probabilistic estimate gives the following distribution (in millions of dollars):

Profit	1	1.5	2	4	10
Probability	.1	.2	.4	??	.1

a. Find  $P(X = 4)$

$$= .2$$

$$.1 + .2 + .4 + x + .1 = 1$$

$$.8 + x = 1$$

$$x = .2$$

b. Find the mean profit and the standard deviation of the profit.

$$E[X] = 1(.1) + 1.5(.2) + 2(.4) + 4(.2) + 10(.1)$$

$$E[X] = 3$$

$$\star \text{VAR}[X] = E[X^2] - (E[X])^2$$

$$\star \text{sd dev} = \sqrt{\text{VAR}[X]} = 2.52$$

c. Gabelli Partners owes its source of capital a fee of \$200,000 plus 10% of the profits X. So the firm actually retains  $Y = .9X - 0.2$  from the investment. Find the mean and standard deviation of Y.

$$E[Y] = E[.9X - .2] = .9E[X] - .2 = 2.5$$

$$\text{Var}[Y] = \text{Var}[.9X - .2] = .9^2 \text{Var}[X] = 5.1435$$

$$\text{sd} = \sqrt{5.1435} = 2.268$$

$$E[x^2] = (1)^2(.1) + (1.5)^2(.2) + (2)^2(.4) + (4)^2(.2) + (10)^2(.1) \\ = 15.35$$

$$E[x] = 3 \rightarrow (E[x])^2 = 9$$

$$\text{Var}[x] = 15.35 - 9 = 6.35$$

$$\text{sd.} = \sqrt{6.35} = 2.52$$

29. The following table displays the results of a sample of 99 in which the subjects indicated their favorite sport of three listed. The data are organized by favorite sport and age group.

Age	Football	Baseball	Soccer
Over 40	15	8	7
20 - 40	20	11	15
Under 20	8	7	8

What is the probability that a person chosen at random will be over 40 if he or she favors football?

$$P(\text{Over 40} | \text{Football}) = \frac{15}{43}$$

30. The probabilities that a customer selects 1, 2, 3, 4, or 5 items at a convenience store are 0.32, 0.12, 0.23, 0.18, and 0.15, respectively.

- a. Construct a probability distribution for the data

X	1	2	3	4	5
P(X)	.32	.12	.23	.18	.15

b. Find  $P(X > 3.5) = P(X = 4) + P(X = 5) = .18 + .15 = .33$

c. Find  $P(1.0 < X < 3.0) = P(X = 2) = .12$

d. Find  $P(X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$   
 $= 1 - P(X = 5) = 1 - P(X \geq 5) = 1 - .15 = .85$

$$P(X < a) = 1 - P(X \geq a)$$

$$P(X > a) = 1 - P(X \leq a)$$

$$P(X \leq a) = P(X < a+1) = 1 - P(X \geq a+1)$$

31. A psychologist interested in right-handedness versus left-handedness and in IQ scores collected the following data from a random sample of 2000 high school students.

	Right-handed	Left-handed	Total
High IQ	190	10	200
Normal IQ	1710	90	1800
Total	1900	100	2000

- a. What is the probability that a student from this group has a high IQ?  $\frac{200}{2000} = .1$

- b. What is the probability that a student has a high IQ given that she is left-handed?

$$P(HIQ | L) = 10/100 = .1$$

$$P(HIQ \cap L) / P(L) = \frac{10/2000}{100/2000} = .1$$

- c. Are high IQ and left-handed independent? Why or why not?

$$P(HIQ | L) = P(HIQ) \quad \text{yes}$$

32. What is the expected value of the discrete probability function given in the table below?

Outcome	1	2	3	4	5	6
Probability	.1	.2	.3	.3	0	.1

$$E[x] = 1(.1) + 2(.2) + 3(.3) + 4(.3) + 5(0) + 6(.1) = 3.2$$



33. A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

a. What kind of distribution does  $X$  have? (Binomial or Geometric?)

$$p = .8 \quad n = 100 \quad \text{Binomial}$$

$p$  same each trial  
indep. trials  
fixed # trials

b. Calculate the mean and standard deviation of  $X$ .

$$\mu_X = 100(.8) = 80 \quad \sigma_X = \sqrt{100(.8)(.2)} = 4$$

c. Determine the probability that exactly 80 subjects experience headache relief with this remedy.

$$P(X=80) = .0993$$

d. What is the probability that between 75 and 90 (inclusive) of the patients will obtain relief?

$$P(75 \leq X \leq 90) = P(X \leq 90) - P(X \leq 74) = .9102$$

$$P(X \leq a) = \text{binomialcdf}(n, p, a)$$

$$P(X \leq a) = \text{pbinom}(a, n, p)$$

$$P(X < a) = P(X \leq a-1)$$

$$P(b \leq X \leq a) = P(X \leq a) - P(X \leq b-1)$$

34. A basketball player completes 64% of her free-throws. We want to observe this player during one game to see how many free-throw attempts she makes before completing one.

a. What type of distribution is this?

Geometric

b. What is the probability that the player misses 3 free-throws before she has makes one?

.0299      1st success on 4<sup>th</sup> attempt  $P(X=4)$   
 $(1-p)^3(p)$

c. How many free-throw attempts can the player expect to throw before she gets a basket?

$$E[X] = \mu = \frac{1}{p} = 1.5625$$

d. Determine the probability that it takes more than 5 attempts before she makes a basket.

$$P(X > 5) = (1-p)^5 = (1-.64)^5$$

$$= \underline{\underline{.006}}$$

$n = 1^{\text{st}} \text{ Success}$

Geom. {

$$\begin{cases} P(X=n) = (1-p)^{n-1} p \\ P(X > n) = (1-p)^n \\ E[X] = \mu = \frac{1}{p} \\ \sigma^2 = \frac{1-p}{p^2} \end{cases}$$