Math 3339  
Homework 1 (Chapter 1 & Chapter 3) 

Name:__________________________________ PeopleSoft ID:_______________

Instructions:
• Homework will NOT be accepted through email or in person. Homework must be submitted through CourseWare BEFORE the deadline.
• Print out this file use or software and complete the problems.
• Write in black ink or dark pencil or type your solutions in the space provided. You must show all work for full credit.
• Submit this assignment at [http://www.casa.uh.edu](http://www.casa.uh.edu) under “Assignments” and choose hw1.
• Total possible points: 15

1. Section 1.9, Problem 1.

   The variables are: NOISE SIZE TYPE SIDE

   Noise – numeric (continuous)
   Size – factor
   Type – factor
   Side – factor

2. Section 1.9 problem 7.

   A random experiment consists of tossing a coin 4 times. Describe the sample space of this experiment. In what proportion of all outcomes of the experiment will there be exactly 2 heads?

   \[
   \Omega = \{x_i | x_i = \text{(four tosses that result in a heads or tails)} \} \quad i = 1, \ldots, n, \quad n = 2^4 = 16 \quad \text{total outcomes}
   \]

   Number of times exactly 2 heads are up: \(4C2 = \text{choose}(4,2) = 6\)

   Proportion of exactly 2 heads up: \(6/16 = 0.375\)
3. Section 3.3.1 problem 7

```r
> dim(mammals)
[1] 62  2
> sum(body>200)
[1] 7
> 7/62
[1] 0.1129032
```

4. Section 3.5.1
   a. Problem 2
   b. Problem 3

   a. A hand of 5-card draw poker is a simple random sample from the standard deck of 52 cards. How many 5 draw poker hands are there? In 5-card stud poker, the cards are dealt sequentially and the order of appearance is important. How many 5-stud poker hands are there?

   5 draw poker: \( \binom{52}{5} = \text{choose}(52,5) = 2598960 \)
   5-stud poker: \( \frac{52!}{(52-5)!} = \frac{52!}{47!} = 311875200 \)

   b. How many hands of 5-draw poker contain the ace of hearts? What is the probability that a 5-card draw hand contains the ace of hearts?

   To calculate the number of hands of 5-draw poker that contain the ace of hearts note that once we know that a hand contains the ace of hearts, we must choose four more cards from 51 remaining cards. \( N(\text{hands contains ace of hearts}) = 1 \times \binom{51}{4} = \text{choose}(51,4) = 249900 \)

   \[ P(\text{5-card hand contains ace of hearts}) = \frac{249900}{2598960} = 0.0962 \]
Section 3.5.1

a. Problem 4
b. Problem 5

a. Everybody in Ourtown is a fool or a knave or possibly both. 70% of the citizens are fools and 85% are knaves. One citizen is randomly selected to be mayor. What is the probability that the mayor is both a fool and a knave?

Since everyone is either a fool (F) or a knave (K) then, \( P(K \cup F) = 1 \), and by the addition rule

\[
1 = P(F) + P(K) - P(K \cap F)
\]

\[
1 = 0.7 + 0.85 - P(K \cap F)
\]

\[
P(K \cap F) = 0.55
\]

b. What is the probability that the mayor is a fool but not a knave?

\[
P(F \sim K) = P(F) - P(K \cap F) = 0.7 - 0.55 = 0.15
\]
6. A box in a certain supply room contains four 40-watt light bulbs, five 60-watt bulbs, and six 75-watt bulbs. Suppose that three bulbs are randomly selected.
   a. What is the probability that exactly two of the selected bulbs are rated 75-watt?
   b. What is the probability that all three of the selected bulbs have the same rating?
   c. What is the probability that one bulb of each type is selected?
   d. What is the probability that at least two of the selected bulbs are rated 75-watt?

4 40-w
5 60-w
6 75-w
15 bulbs

   a. \( P(\text{exactly 2 out of 3 are 75-w}) = \frac{\binom{6}{2} \cdot \binom{9}{1}}{\binom{15}{3}} = 0.2967 \)
   b. \( P(\text{all 3 have the same rating}) = P(3\ 40\text{-w or } 3\ 60\text{-w or } 3\ 75\text{-w}) \)
      \[= \frac{\binom{4}{3} + \binom{5}{3} + \binom{6}{3}}{\binom{15}{3}} = 0.0747 \]
   c. \( P(\text{1 bulb of each type is selected}) = \frac{4 \cdot 5 \cdot 6}{\binom{15}{3}} = 0.2637 \)
   d. \( P(\text{at least two out of 3 are 75-w}) = P(2 \text{ or } 3 \ 75\text{-w}) = 0.2967 + \frac{\binom{6}{3}}{\binom{15}{3}} = 0.3407 \)
7. A certain system can experience three different types of defects. Let \( A_i (i = 1, 2, 3) \) denote the event that the system has a defect of type \( i \). Suppose that:

\[
\begin{align*}
P(A_1) &= 0.12; \ P(A_2) = 0.07; \ P(A_3) = 0.05 \\
P(A_1 \cup A_2) &= 0.13; \ P(A_1 \cup A_3) = 0.14; \ P(A_2 \cup A_3) = 0.10 \\
P(A_1 \cap A_2 \cap A_3) &= 0.01
\end{align*}
\]

a. What is the probability that the system does not have a type 1 defect?
b. What is the probability that the system has both type 1 and type 2 defects?
c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?
d. What is the probability that the system has at most two of these defects?

\[
\begin{align*}
a. \ P(\sim A_1) &= 1 - 0.12 = 0.88 \\
b. \ P(A_1 \cap A_2) &= 0.12 + 0.07 - 0.13 = 0.06 \\
c. \ P(A_1 \cap A_2 \cap \sim A_3) &= 0.06 - 0.01 = 0.05 \\
d. \ 1 - 0.01 = 0.99
\end{align*}
\]

To answer these questions it will help to use the Venn Diagram

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*a Modern Mathematical Statistics with Applications*, Devore, J. and Berk, K., Thomson (2007), Chapter 2
8. Consider randomly selecting a student at a certain university, and let $A$ denote the event that the selected individual has a Visa credit card and $B$ be the analogous event for MasterCard. Suppose that $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.25$. Calculate and interpret each of the following probabilities.
   a. $P(B \mid A)$
   b. $P(\neg B \mid A)$
   c. $P(A \mid B)$
   d. $P(\neg A \mid B)$
   e. Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?
   f. Is having a Visa credit card and a MasterCard independent? Justify your answer.

   a. $P(B \mid A) = \frac{0.25}{0.5} = 0.5$
   b. $P(\neg B \mid A) = \frac{P(A \sim B)}{P(A)} = \frac{0.5 - 0.25}{0.5} = 0.5$
   c. $P(A \mid B) = \frac{0.25}{0.4} = 0.625$
   d. $P(\neg A \mid B) = \frac{P(B \sim A)}{P(B)} = \frac{0.4 - 0.25}{0.4} = 0.375$
   e. $P(A \mid A \cup B) = \frac{0.5}{0.65} = 0.7692$
   f. No, since $P(A)\cdot P(B) \neq P(A \cap B)$
9. Suppose two events $A$ and $B$ are two independent events with $P(A) > P(B)$ and $P(A \cup B) = 0.626$ and $P(A \cap B) = 0.144$, determine the values of $P(A)$ and $P(B)$.

We can use both the addition rule and multiplication rule to determine $P(A)$ and $P(B)$.

Since independent $P(A \cap B) = P(A)P(B)$
Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.626 = P(A) + P(B) - 0.144 \rightarrow P(A) + P(B) = 0.77$
P(A)*P(B) = 0.144

$P(A) = 0.45; P(B) = 0.32$
10. Suppose that out of 20% of all packages from Amazon are delivered by UPS. 12% of the packages that are delivered by UPS weighs 2 lbs or more. Also, 8% of the packages that are not delivered by UPS weighs less than 2 lbs.

   a. What is the probability that a package is delivered by UPS if it weighs 2 lbs or more?

   b. What is the probability that a package is not delivered by UPS if it weighs 2 lbs or more?

Let \( A = \) packages delivered by UPS. Let \( B = \) packages that weight 2 lbs or more.

\[
P(A) = 0.2; \quad P(B \mid A) = 0.12; \quad P(B \mid \neg A) = 0.08
\]

Using the following tree diagram can help solve this problem

\[
P(A \cap B) = 0.2 \times 0.12 = 0.024
\]

\[
P(A \cap \neg B) = 0.2 \times 0.88 = 0.176
\]

\[
P(\neg A \cap B) = 0.8 \times 0.92 = 0.736
\]

\[
P(\neg A \cap \neg B) = 0.8 \times 0.08 = 0.064
\]

a. \[P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.024}{0.024 + 0.736} = 0.0316\]

b. \[P(\neg A \mid B) = 1 - 0.0316 = 0.9684\]