Popular Discrete and Continuous Distributions

Bayes' Theorem

$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{P(B)} = \frac{P(B \mid A_j)P(A_j)}{\sum_i P(B \mid A_i)P(A_i)}$$

Binomial(n,p)

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots$$

$$E[X] = np \quad Var[X] = np(1-p)$$

Poisson(λ)

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,\ldots$$

$$E[X] = \lambda \quad Var[X] = \lambda$$

NegativeBinomial(r,p)

$$f(x) = \binom{x+r-1}{r-1} p^r (1-p)^x, \quad x = 0,1,2,\ldots$$

$$E[X] = \frac{r(1-p)}{p} \quad Var[X] = \frac{r(1-p)}{p^2}$$

Geometric(p)

$$f(x) = p(1-p)^x, \quad x = 0,1,2,\ldots$$

$$E[X] = \frac{1-p}{p} \quad Var[X] = \frac{1-p}{p^2} \quad M_X(t) = \frac{p}{1-(1-p)e^t}, \; t \in \mathbb{R}, (1-p)e^t < 1$$

Hypergeometric

$$f(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N_1+N_2}{n}}, \quad x = 0,1,2,\ldots,n$$

Gamma(α,β)

$$f(x)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta},\quad 0\le x<\infty \qquad \text{Note: } \Gamma(r)=(r-1)!, \text{ for all } r=1,2,3,\ldots$$

$$E[X]=\alpha\beta \quad Var[X]=\alpha\beta^{2}$$

Normal(μ,σ)

$$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(x-\mu)^2}{2\sigma^2}},\ x\in\mathbb{R}$$

$$E[X]=\mu \quad Var[X]=\sigma^{2}$$

Exponential (λ)

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & otherwise\end{cases}$$

$$E[X]=1/\lambda$$

$$Var[X]=1/\lambda^{2}$$

R “base” commands for distributions:

“_” filled in with d, p or q

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_binom( )
_exp( )
_gamma( )
_hyper( )
_nbinom( )
_norm( )
_pois( )
_weibull( )
_t( )
_f( )
_tukey( )
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Formulas for joint distributions:

$$P(Y=y \mid X=x) = \frac{P(X=x, Y=y)}{P(X=x)} = \frac{p(x,y)}{p_X(x)}$$

$$P(X=x \mid Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{p(x,y)}{p_Y(y)}$$

$$\mu_X = E[X] = \sum_{(x,y)\in {}^\circ} x\, p(x,y) = \sum_x x\, p_X(x)$$

$$\mu_Y = E[Y] = \sum_{(x,y)\in {}^\circ} y\, p(x,y) = \sum_y y\, p_Y(y)$$

$$E\left[g(X,Y)\right] = \sum_{(x,y)\in {}^\circ} g(x,y)\, p(x,y)$$

$$\operatorname{cov}(X,Y) = E[XY] - E[X]E[Y]$$

$$\rho = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

Hypothesis tests and confidence intervals:

Test	**Null Hypothesis**	**Test Statistic**
One-sample z-test for means	$\mu = \mu_o$	$z = \dfrac{\bar{x} - \mu_o}{\frac{\sigma}{\sqrt{n}}}$
One-sample t-test for means	$\mu = \mu_o$	$t = \dfrac{\bar{x} - \mu_o}{\frac{s}{\sqrt{n}}}$; df = n–1
Matched Pairs t-test	$\mu_D = \mu_{D_0}$	$t = \dfrac{\bar{w} - \mu_{D_0}}{s/\sqrt{n}}$; df = n – 1
One-sample z-test for proportions	$p = p_o$	$z = \dfrac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

Two-sample t-test for means	$\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$	$t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$; df=v $v = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$
Two-sample z-test for means	$\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$	$z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ or $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$
Two-sample z-test for proportions	$p_1 - p_2 = 0$ or $p_1 = p_2$	$z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$
χ^2 - Goodness of Fit	__ is same as _	$\chi^2 = \sum \dfrac{(O-E)^2}{E}$

Confidence Intervals

One-sample z-test: $\bar{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$

One-sample t-test: $\bar{x} \pm t^* \dfrac{s}{\sqrt{n}}$

One-proportion z-test: $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Confidence interval for σ^2: $\left[\dfrac{(n-1)S^2}{\chi^2_{\alpha/2}}, \dfrac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right]$

χ^2 has $n-1$ degrees of freedom.

Two-sample t-test: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

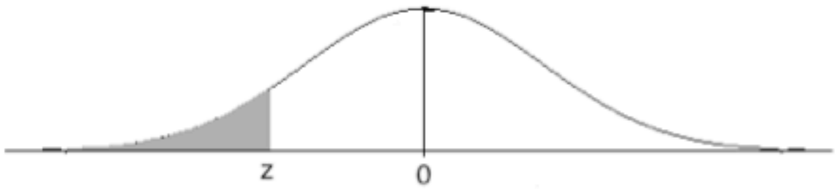
Two-sample z-test: $(\bar{x}_1 - \bar{x}_2) \pm z^* \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Tukey's w: $w = Q_{\alpha, M, N-M} \sqrt{\dfrac{MS(resid)}{N/M}}$

Two-proportion z-test: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

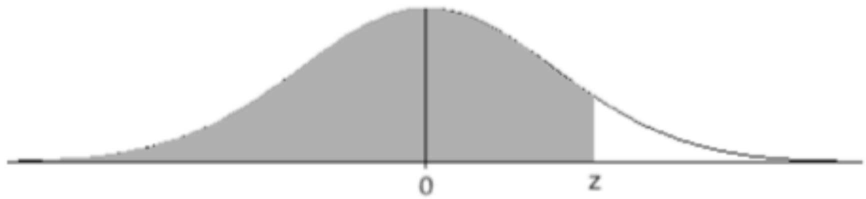
Slope of regression line: $(\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x)$: $\hat{\beta}_1 \pm t_{\alpha/2} SE_{\hat{\beta}_1}$

Table of Standard Normal Probabilities for Negative Z-scores



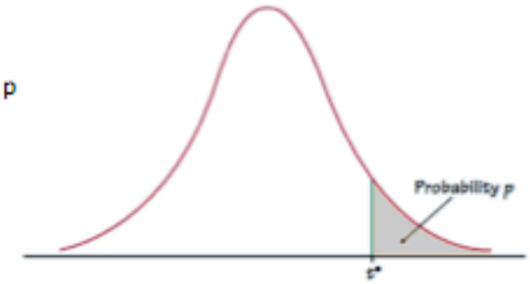
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



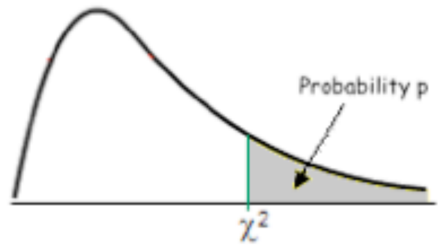
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table entry for p and C is the critical value t* with probability p lying to its right and probability C lying between -t* and t*



	Upper tail probability *p*											
df	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.0025**	**0.001**	**0.0005**
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	4.8487	6.9646	9.9248	14.0890	22.3271	31.5991
3	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	3.4819	4.5407	5.8409	7.4533	10.2145	12.9240
4	0.7407	0.9410	1.1896	1.5332	2.1318	2.7764	2.9985	3.7469	4.6041	5.5976	7.1732	8.6103
5	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	2.7565	3.3649	4.0321	4.7733	5.8934	6.8688
6	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	2.6122	3.1427	3.7074	4.3168	5.2076	5.9588
7	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.5168	2.9980	3.4995	4.0293	4.7853	5.4079
8	0.7064	0.8889	1.1081	1.3968	1.8595	2.3060	2.4490	2.8965	3.3554	3.8325	4.5008	5.0413
9	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.3984	2.8214	3.2498	3.6897	4.2968	4.7809
10	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.3593	2.7638	3.1693	3.5814	4.1437	4.5869
11	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.3281	2.7181	3.1058	3.4966	4.0247	4.4370
12	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.3027	2.6810	3.0545	3.4284	3.9296	4.3178
13	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.2816	2.6503	3.0123	3.3725	3.8520	4.2208
14	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.2638	2.6245	2.9768	3.3257	3.7874	4.1405
15	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.2485	2.6025	2.9467	3.2860	3.7328	4.0728
16	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208	3.2520	3.6862	4.0150
17	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.2238	2.5669	2.8982	3.2224	3.6458	3.9651
18	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.2137	2.5524	2.8784	3.1966	3.6105	3.9216
19	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.2047	2.5395	2.8609	3.1737	3.5794	3.8834
20	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.1967	2.5280	2.8453	3.1534	3.5518	3.8495
21	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.1894	2.5176	2.8314	3.1352	3.5272	3.8193
22	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.1829	2.5083	2.8188	3.1188	3.5050	3.7921
23	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.1770	2.4999	2.8073	3.1040	3.4850	3.7676
24	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.1715	2.4922	2.7969	3.0905	3.4668	3.7454
25	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.1666	2.4851	2.7874	3.0782	3.4502	3.7251
26	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.1620	2.4786	2.7787	3.0669	3.4350	3.7066
27	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.1578	2.4727	2.7707	3.0565	3.4210	3.6896
28	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.1539	2.4671	2.7633	3.0469	3.4082	3.6739
29	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.1503	2.4620	2.7564	3.0380	3.3962	3.6594
30	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.1470	2.4573	2.7500	3.0298	3.3852	3.6460
40	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.1229	2.4233	2.7045	2.9712	3.3069	3.5510
50	0.6794	0.8489	1.0473	1.2987	1.6759	2.0086	2.1087	2.4033	2.6778	2.9370	3.2614	3.4960
60	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.0994	2.3901	2.6603	2.9146	3.2317	3.4602
80	0.6776	0.8461	1.0432	1.2922	1.6641	1.9901	2.0878	2.3739	2.6387	2.8870	3.1953	3.4163
100	0.6770	0.8452	1.0418	1.2901	1.6602	1.9840	2.0809	2.3642	2.6259	2.8707	3.1737	3.3905
1000	0.6747	0.8420	1.0370	1.2824	1.6464	1.9623	2.0564	2.3301	2.5808	2.8133	3.0984	3.3003
z*	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.0537	2.3263	2.5758	2.8070	3.0902	3.2905
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence Level *C*											

Table entry for p is the critical value χ^2 with probability p lying to its right.



df	Upper tail probability *p*											
df	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.0025**	**0.001**	**0.0005**
1	1.323	1.642	2.072	2.706	3.841	5.024	5.412	6.635	7.879	9.141	10.828	12.116
2	2.773	3.219	3.794	4.605	5.991	7.378	7.824	9.210	10.597	11.983	13.816	15.202
3	4.108	4.642	5.317	6.251	7.815	9.348	9.837	11.345	12.838	14.320	16.266	17.730
4	5.385	5.989	6.745	7.779	9.488	11.143	11.668	13.277	14.860	16.424	18.467	19.997
5	6.626	7.289	8.115	9.236	11.070	12.833	13.388	15.086	16.750	18.386	20.515	22.105
6	7.841	8.558	9.446	10.645	12.592	14.449	15.033	16.812	18.548	20.249	22.458	24.103
7	9.037	9.803	10.748	12.017	14.067	16.013	16.622	18.475	20.278	22.040	24.322	26.018
8	10.219	11.030	12.027	13.362	15.507	17.535	18.168	20.090	21.955	23.774	26.124	27.868
9	11.389	12.242	13.288	14.684	16.919	19.023	19.679	21.666	23.589	25.462	27.877	29.666
10	12.549	13.442	14.534	15.987	18.307	20.483	21.161	23.209	25.188	27.112	29.588	31.420
11	13.701	14.631	15.767	17.275	19.675	21.920	22.618	24.725	26.757	28.729	31.264	33.137
12	14.845	15.812	16.989	18.549	21.026	23.337	24.054	26.217	28.300	30.318	32.909	34.821
13	15.984	16.985	18.202	19.812	22.362	24.736	25.472	27.688	29.819	31.883	34.528	36.478
14	17.117	18.151	19.406	21.064	23.685	26.119	26.873	29.141	31.319	33.426	36.123	38.109
15	18.245	19.311	20.603	22.307	24.996	27.488	28.259	30.578	32.801	34.950	37.697	39.719
16	19.369	20.465	21.793	23.542	26.296	28.845	29.633	32.000	34.267	36.456	39.252	41.308
17	20.489	21.615	22.977	24.769	27.587	30.191	30.995	33.409	35.718	37.946	40.790	42.879
18	21.605	22.760	24.155	25.989	28.869	31.526	32.346	34.805	37.156	39.422	42.312	44.434
19	22.718	23.900	25.329	27.204	30.144	32.852	33.687	36.191	38.582	40.885	43.820	45.973
20	23.828	25.038	26.498	28.412	31.410	34.170	35.020	37.566	39.997	42.336	45.315	47.498
21	24.935	26.171	27.662	29.615	32.671	35.479	36.343	38.932	41.401	43.775	46.797	49.011
22	26.039	27.301	28.822	30.813	33.924	36.781	37.659	40.289	42.796	45.204	48.268	50.511
23	27.141	28.429	29.979	32.007	35.172	38.076	38.968	41.638	44.181	46.623	49.728	52.000
24	28.241	29.553	31.132	33.196	36.415	39.364	40.270	42.980	45.559	48.034	51.179	53.479
25	29.339	30.675	32.282	34.382	37.652	40.646	41.566	44.314	46.928	49.435	52.620	54.947
26	30.435	31.795	33.429	35.563	38.885	41.923	42.856	45.642	48.290	50.829	54.052	56.407
27	31.528	32.912	34.574	36.741	40.113	43.195	44.140	46.963	49.645	52.215	55.476	57.858
28	32.620	34.027	35.715	37.916	41.337	44.461	45.419	48.278	50.993	53.594	56.892	59.300
29	33.711	35.139	36.854	39.087	42.557	45.722	46.693	49.588	52.336	54.967	58.301	60.735
30	34.800	36.250	37.990	40.256	43.773	46.979	47.962	50.892	53.672	56.332	59.703	62.162
40	45.616	47.269	49.244	51.805	55.758	59.342	60.436	63.691	66.766	69.699	73.402	76.095
50	56.334	58.164	60.346	63.167	67.505	71.420	72.613	76.154	79.490	82.664	86.661	89.561
60	66.981	68.972	71.341	74.397	79.082	83.298	84.580	88.379	91.952	95.344	99.607	102.695
80	88.130	90.405	93.106	96.578	101.879	106.629	108.069	112.329	116.321	120.102	124.839	128.261
100	109.141	111.667	114.659	118.498	124.342	129.561	131.142	135.807	140.169	144.293	149.449	153.167

Table A.4 The Incomplete Gamma Function

$$F(x;\alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y}\, dy$$

x \ α	1	2	3	4	5	6	7	8	9	10
1	.632	.264	.080	.019	.004	.001	.000	.000	.000	.000
2	.865	.594	.323	.143	.053	.017	.005	.001	.000	.000
3	.950	.801	.577	.353	.185	.084	.034	.012	.004	.001
4	.982	.908	.762	.567	.371	.215	.111	.051	.021	.008
5	.993	.960	.875	.735	.560	.384	.238	.133	.068	.032
6	.998	.983	.938	.849	.715	.554	.394	.256	.153	.084
7	.999	.993	.970	.918	.827	.699	.550	.401	.271	.170
8	1.000	.997	.986	.958	.900	.809	.687	.547	.407	.283
9		.999	.994	.979	.945	.884	.793	.676	.544	.413
10		1.000	.997	.990	.971	.933	.870	.780	.667	.542
11			.999	.995	.985	.962	.921	.857	.768	.659
12			1.000	.998	.992	.980	.954	.911	.845	.758
13				.999	.996	.989	.974	.946	.900	.834
14				1.000	.998	.994	.986	.968	.938	.891
15					.999	.997	.992	.982	.963	.930