Math 3339
Review for Test 1 KEY

1. $X \sim binomial(6,.125);\quad P(X=1)=0.3847$

2. $X \sim binomial(10,.85);\quad P(X=8)+P(X=9)+P(X=10)=0.8202$

3. $X \sim binomial(5,.325);\quad P(X \geq 1)=1-P(X<1)=1-P(X=0)=0.8599$

4. $E[X]=18$

5. $E[X]=3.7\ miles,\ V[X]=2.61$

6. $E[X]=2,\ V[X]=1.2$

7. $\tilde{\mu}=3.65$

8. $median=172,\ \bar{x}=169.4,\ s^2=52.30$

9. $X \sim binomial(10,.6);\quad P(X \geq 5)=0.8338$

10.

#11 The test grades for a certain class were entered into a Minitab worksheet, and then Descriptive Statistics were requested. The results were:

MTB> Describe 'Grades'.

	N	MEAN	MEDIAN (Q2)	TRMEAN	STDEV	SEMEAN
Grades	28	74.71	76.00	75.50	12.61	2.38

	MIN	MAX	Q1	Q3
Grades	35.00	94.00	68.00	84.00

You happened to see, on a scrap of paper, that the lowest grades were 35, 57, 59, 60, … but you don't know what the other individual grades are. Nevertheless, a knowledgeable user of statistics can tell a lot about the data set simply by studying the set of descriptive statistics above.

a. Write a brief description of what the results in the box tell you about the distribution of grades. Be sure to address:

   i. The general shape of the distribution slightly skewed left
   ii. Unusual features, including possible outliers
   iii. The middle 50% of the data $IQR = 16$ (68,84)
   iv. Any significance in the difference between the mean and the median not really

$IQR = Q3 - Q1 = 16$

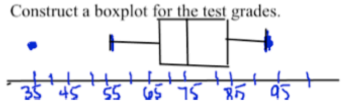
$1.5\,IQR = 24$

outliers $Q1-24,\ Q3+24$

(44, 108)

35 is an outlier

b. Construct a boxplot for the test grades.



11. 0.33

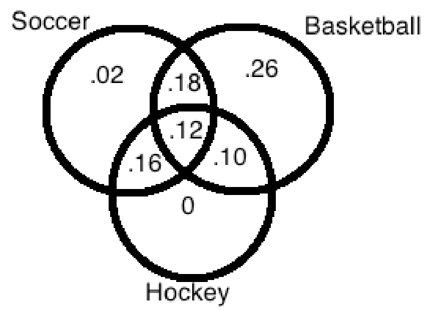
12.

a. 0.1
b. 0.1
c. yes $P(H \mid L) = P(H)$

13.

a. 0.027
b. 0.778

14.



a.
b. 0.82
c. 0.16

15. 0.5

16.

a. $P(X = 4) = 0.15$
b. $P(1 \le X < 3) = 0.45$
c. 2.25
d. 1.178
e. 3

17.

a. $E[Y] = 57$
b. $V[Y] = 17.64$
c. 4.2
d. 17.64

18.

a. 192
b. 108
c. 0.4375 or 7/16

19.

a. $P(A|B)=\frac{2}{3}$

b. No

20.

a.

y\x	1	2	
1	4/39	7/39	11/39
2	5/39	8/39	13/39
3	6/39	9/39	15/39
	15/39	24/39	1

b. (in red on table)

$E[X]=\frac{21}{13}$

c. $E[Y]=\frac{82}{39}$

$E[XY]=\frac{132}{39}$

$\sigma_X=0.4865$

d. $\sigma_Y=0.8100$

$\text{cov}(X,Y)=-0.012$

e. Not independent

21.

a. 0.044

b. 0.3409

22. 22

23. 1-phyper(1,20,15,5)=1 - 0.0933457 = 0.9066543

24. 0.1804

25. Suppose that from a group of 9 men and 8 women, a committee of 5 people is to be chosen.

a. What type of probability distribution is this?

Hypergeometric with m = 9, n = 8 and k = 5

b. What is the probability that the committee has exactly 3 men and 2 women?

$$\frac{\binom{9}{3}\binom{8}{2}}{\binom{17}{5}}=\frac{84\times 28}{6188}=0.3801$$

26. A restaurant serves three fixed-price dinners costing \$12, \$15, and \$20. For a randomly selected couple dining at this restaurant, let X = the cost of the man's dinner and Y = the cost of the woman's dinner. The joint pmf of X and Y is given in the following table:

$P(x, y)$		Y = 12	15	20
	12	0.05	0.05	0.10
X	15	0.05	0.10	0.35
	20	0	0.20	0.10

a. Compute the marginal pmf's of X and Y.

Marginal pmf of X

X	12	15	20
P(X)	0.05 + 0.05 + 0.10 = 0.20	0.05 + 0.10 + 0.35 = 0.5	0 + 0.2 + 0.1 = 0.3

Marginal pmf of Y

Y	12	15	20
P(Y)	0.05 + 0.05 + 0 = 0.1	0.05 + 0.10 + 0.20 = 0.35	0.1 + 0.35 + 0.1 = 0.55

b. What is the probability that the man's and the woman's dinner cost at most \$15 each?

$P(X \leq 15 \text{ and } Y \leq 15) = P(X = 12, Y = 12) + P(X = 12, Y = 15) + P(X = 15, Y = 12) + P(X = 15, Y = 15) = 0.05 + 0.05 + 0.05 + 0.10 = 0.25$

c. Are X and Y independent? Justify your answer.

X	12	15	20
P(X | Y = 12)	0.05/0.1 = 0.5	0.05/0.1 = 0.5	0/0.1 = 0

Since this conditional probability is not the same as the probability distribution of X, X and Y are **not independent**.

d. What is the expected value of the total cost of the dinner for the two people?

$E(X) = 12(0.2) + 15(0.5) + 20(0.3) = 2.4 + 7.5 + 6 = 15.9$

$E(Y) = 12(0.1) + 15(0.35) + 20(0.55) = 1.2 + 5.25 + 11 = 17.45$

$E(X + Y) = 15.9 + 17.45 =$ \$33.35

e. Suppose the when a couple opens fortune cookies at the conclusion of the meal, they find the message "You will receive as a refund the difference between the cost of the more expensive and the less expensive meal that you have chosen." How much does the restaurant expect to refund?

$|E(X – Y)| = |E(X) – E(Y)| = |15.9 – 17.45| = |-1.55| =$ \$1.55

27. The weight of a randomly selected bag of corn chips coming off an assembly line is a random variable with mean μ= 10 oz. and standard deviation σ= 0.2 oz. Suppose we pick four bags at random assume that weight of each of the bags are independent.

Let X_1, X_2, X_3, X_4 be the weights of each of the four bags.

a. What is the mean of the combined weight of these four bags?

We want $E(X_1 + X_2 + X_3 + X_4) = 10 + 10 + 10 + 10 = 40$

b. What is the standard deviation of the combined weight of these four bags?

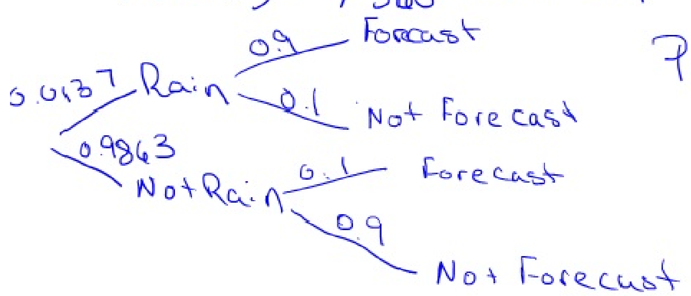
$Var(X_1 + X_2 + X_3 + X_4) = 0.04 + 0.04 + 0.04 + 0.04 = 0.16$

$SD((X_1 + X_2 + X_3 + X_4) = \sqrt{0.16} = 0.4$

28. Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding, given the weatherman forecasts rain?

Let A = a rainy day, B = weatherman forecasts rain.
P(A) = 5/365 = 0.0137
P(B | A) = 0.9
P(~B | ~ A) = 0.1

$$P(\text{Rain}) = 5/365 = 0.0137$$



$$P(\text{Rain} \mid \text{Forecast}) = \frac{0.0137(0.9)}{0.0137(0.9) + 0.9863(0.1)} = \frac{0.01233}{0.11096} = 0.1111$$