1. \textit{Munkres, Exercise 26.2.} \\
(a) Show that in the finite complement topology on \(\mathbb{R}\), every subspace is compact. \\
(b) If \(\mathbb{R}\) has the topology consisting of all sets \(A\) such that \(\mathbb{R} \setminus A\) is either countable or all of \(\mathbb{R}\), is \([0, 1]\) a compact subspace?

2. \textit{Munkres, Exercise 26.11.} Prove the following theorem. Let \(X\) be a compact Hausdorff space. Let \(\mathcal{A}\) be a collection of closed connected subsets of \(X\) that is simply ordered by proper inclusion. Then \(Y = \bigcap_{A \in \mathcal{A}} A\) is connected. \\
\textit{Hint:} If \(C \cup D\) is a separation of \(Y\), choose disjoint open sets \(U\) and \(V\) of \(X\) containing \(C\) and \(D\), respectively, and show that \(\bigcap_{A \in \mathcal{A}} (A \setminus (U \cup V))\) is not empty.

3. \textit{Munkres, Exercise 26.12.} Let \(p: X \to Y\) be a closed continuous surjective map such that \(p^{-1}(\{y\})\) is compact for every \(y \in Y\). (Such a map is called a \textit{perfect map.}) Show that if \(Y\) is compact, then \(X\) is compact. \\
\textit{Hint:} If \(U\) is an open set containing \(p^{-1}(\{y\})\), then there is a neighbourhood \(W\) of \(y\) such that \(p^{-1}(W)\) is contained in \(U\).

4. \textit{Munkres, Exercise 27.3.} Recall that \(\mathbb{R}_K\) denotes \(\mathbb{R}\) in the \(K\)-topology. \\
(a) Show that \([0, 1]\) is not compact as a subspace of \(\mathbb{R}_K\). \\
(b) Show that \(\mathbb{R}_K\) is connected. \textit{Hint:} \((-\infty, 0)\) and \((0, \infty)\) inherit their usual topologies as subspaces of \(\mathbb{R}_K\). \\
(c) Show that \(\mathbb{R}_K\) is not path connected.

5. \textit{Munkres, Exercise 27.5.} Let \(X\) be a compact Hausdorff space; let \(\{A_n\}\) be a countable collection of closed sets of \(X\). Show that if each set \(A_n\) has empty interior in \(X\), then the union \(\bigcup A_n\) has empty interior in \(X\). \textit{Hint:} imitate the proof of Theorem 27.7. This is a special case of the Baire category theorem.