CORRECTIONS TO "FROM GROUPS TO GEOMETRY AND BACK"

VAUGHN CLIMENHAGA

I am grateful to Lewis Robinson for alerting me to the following errors.

- Page 87: In Figure 2.7, the labels d_2 and d_3 in the left-hand picture should be exchanged.
- Page 160: At the end of the second-to-last paragraph, the characteristic polynomial of the rotation matrix is missing a factor of λ in its second term. The correct formula is $p(\lambda) = \lambda^2 - 2(\cos\theta)\lambda + 1$.
- Page 168: In Definition 3.8, the definition of *transversal* should have $W' \cap W = \{\mathbf{0}\}$ instead of $W \cap W = \{\mathbf{0}\}$.
- Page 178: On the last line of the page, "$\mathbf{x}$ and $-\mathbf{x}$" should be corrected to read "$\mathbf{p}$ and $-\mathbf{p}$".

A further correction was pointed out to me by Yago Antolín, to whom I am also grateful: There is an error at the top of page 296, in the claim that $\mathbb{Z}^2$ appears in the group $H = \langle a, b, c \mid abca^{-1}b^{-1}c^{-1} = e\rangle$ as a subgroup of index two generated by $A = ab$ and $B = cb$. This claim is false, as it would imply that H does not contain any nonabelian free subgroups, contradicting the Freiheitssatz, which in the present setting implies that each of $\langle a, b\rangle$, $\langle a, c\rangle$, and $\langle b, c\rangle$ is nonabelian and free. One can also see the contradiction without invoking the Freiheitssatz by observing that H maps homomorphically onto the free group $\langle g, h\rangle$ by taking $a \mapsto g$, $b \mapsto h$, and $c \mapsto h^{-1}$.

The source of the error in the text is the claim that every word of even length can be written in terms of A and B. This is incorrect; for example, ac admits no such decomposition.

By way of further explanation, observe that the definition of H was motivated by writing down the labels of the boundary of the hexagon and introducing this word as a relation because it is contractible. As the second paragraph of Page 296 explains, the problem is that the symbols a, b, c correspond to paths that are not loops. What is not explicitly stated there is that this in fact renders certain words meaningless if we want to interpret them as paths; for example, ac does not correspond to any path on the surface. Thus the correct approach, which is described earlier on Page 292, is to interpret a, b, c as translations of $\mathbb{R}^2$ rather than as paths, and then the group they generate is isomorphic to $\mathbb{Z}^2$. The group H defined on Page 295 in fact has very little to do with the torus, which was after all the point of the discussion on Page 296.

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