Math 4377/6308 Advanced Linear Algebra I Spring 2013

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MIDTERM TEST #1

Thursday, February 21, 2013

You must give complete justification for all answers in order to receive full credit.

Name: ______________________________

	Points	Possible
Problem 1		/10
Problem 2		/25
Problem 3		/20
Problem 4		/20
Problem 5		/10
Problem 6		/15
Total		/100

1. (a) Let $x \sim y$ be a relation on a set X. Define what it means for $\sim$ to be an *equivalence relation.* [*5 points*]

(b) Define a relation on $\mathbb{R}$ by $x \sim y$ if and only if $xy \geq 0$. Is this an equivalence relation? Prove your answer. [*5 points*]

2. (a) Let $W = \{(x, y) \in \mathbb{R}^2 \mid xy + x = 0\}$. Is W a subspace of $\mathbb{R}^2$? Prove your answer. *[10 points]*

(b) Let V and W be vector spaces and let $T\colon V \to W$ be a linear transformation. Let W_1 be a subspace of W and define a set $V_1 \subset V$ by $V_1 = \{x \in V \mid T(x) \in W_1\}$. Show that V_1 is a subspace of V. *[15 points]*

3. (a) Define what it means for a vector space V to be the direct sum of two subspaces $W_1, W_2 \subset V$. [*5 points*]

(b) In $\mathbb{R}^3$, consider the subspaces $W_1 = \{(0, -b, b) \mid b \in \mathbb{R}\}$ and $W_2 = \{(a_1, a_2 + a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$. Is $\mathbb{R}^3 = W_1 \oplus W_2$? Justify your answer. [*15 points*]

4. (a) Let V be a vector space over a field F, and let $S \subset V$. Define what it means for S to be linearly independent. [*5 points*]

(b) Consider the vector space $\mathbb{P}_3(\mathbb{R})$ consisting of polynomials with degree 3 or less, and the subset

$$S = \{1 + x^3,\ x + x^2,\ x^2 - x^3,\ 1 - x\} \subset \mathbb{P}_3(\mathbb{R}).$$

Does S span $\mathbb{P}_3(\mathbb{R})$? [*15 points*]

5. Let $\{v_1, v_2\}$ be a basis for $\mathbb{R}^2$. Is $\{v_1 + v_2, v_1 - v_2\}$ necessarily a basis for $\mathbb{R}^2$? *[10 points]*

6. (a) Let V be a vector space. State what it means for V to be finite-dimensional. Assuming V is finite-dimensional, state the definition of the dimension of V. [*5 points*]

(b) Let V and W be finite-dimensional vector spaces and $T\colon V \to W$ a linear transformation. Suppose that $\dim W > \dim V$, and prove that T is not onto. [*10 points*]

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