Math 1312

Parallel and Perpendicular Lines

Definitions:

- A **plane** is a two dimensional geometric object**.** It has infinite length and infinite width but no thickness.

- **Parallel lines** are lines that lie in the same plane but do not intersect. (Symbol $\|$)

- **Perpendicular lines** are two lines that meet to form congruent adjacent angles. (Symbol $\perp$)

Example:

Theorem: If two lines are perpendicular, then they meet to form right angles.

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E.
Prove: $\angle AEC$ is a right angle.

PROOF

Statements	**Reasons**
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E	
2. $\angle AEC \cong \angle CEB$	
3. $m\angle AEC = m\angle CEB$	
4. $\angle AEB$ is a straight angle	
5. $m\angle AEC + m\angle CEB = m\angle AEB$	
6. $m\angle AEC + m\angle CEB = 180^0$	
7. $m\angle AEC + m\angle AEC = 180^0$ or $2m\angle AEC = 180^0$	
8. $m\angle AEC = 90^0$	
9. $\angle AEC$ is a right angle	

Definition: The **perpendicular bisector** of a segment is a line (or a segment) that is perpendicular to a given segment and divides it into two congruent segments.

Example:

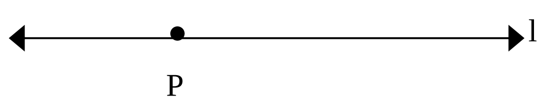
Theorem: The perpendicular bisector of a line segment is unique.

Construction 5: To construct the line perpendicular to a given line through a given point on the line.

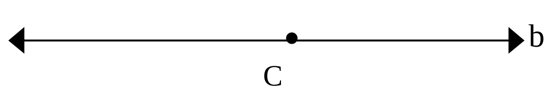
Given: Line l and point P on l.
To construct: $\overleftrightarrow{PQ}$ so that $\overleftrightarrow{PQ} \perp l$.
Construction:

1. Place the compass at the given point P.
2. Draw arcs to the left and right of P, using the same compass setting, and intersecting given line l. Label the points of intersection X and Y.
3. Open the compass to a setting greater than XY.
4. Put the compass at point X and draw an arc above line n.
5. With the same compass setting, repeat the process in #4 at point Y.
6. Label the point of intersection of the two arcs as point Q.
7. Use a straightedge to draw $\overleftrightarrow{PQ}$.

Example:



Try the following:
Construct a line perpendicular to line b and passing through point C.

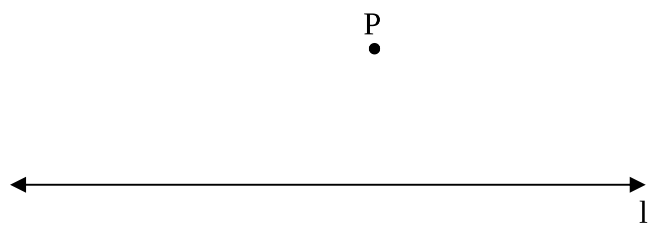


Construction 6: To construct the line perpendicular to a given line through a given point not on the given line.

Given: Line l and point P outside of l.
To construct: $\overleftrightarrow{PQ}$ so that $\overleftrightarrow{PQ} \perp l$.
Construction:

1. Place the compass at the given point P.
2. Draw an arc that intersects the given line l in two different places.
3. Label the points of intersection on line m as X and Y.
4. Open the compass to greater than half of XY.
5. Place the compass at point X and draw and arc below line l.
6. Use the same setting and repeat #5 at point Y.
7. Label the point of intersection of the two arcs as Q.
8. Use a straightedge to draw $\overleftrightarrow{PQ}$.

Example:

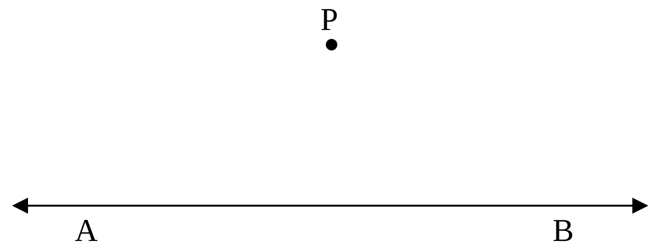


Construction 7: To construct the line parallel to a given line through a given point not on that line.

Given: $\overleftrightarrow{AB}$ and external point P.
To construct: A line through P parallel to $\overleftrightarrow{AB}$.
Construction:

1. Draw a line through point P that intersects line $\overleftrightarrow{AB}$.
2. Label the point of intersection of the new line and line $\overleftrightarrow{AB}$ as point Q.
3. Copy one of the angles that has Q as a vertex. (See Construction #3)
4. Copy it so that P is the vertex and one side of the angle is on line $\overleftrightarrow{PQ}$.
5. Extend the other side of the angle out to form a line.
6. This new line is parallel to the original line $\overleftrightarrow{AB}$

Example:



Relations and Their Properties

Table 1.8 on page 47

Relation	Object Related	Example
Is equal to	numbers	$2 + 3 = 5$
Is greater than	numbers	$7 > 5$
Is perpendicular to	lines	$m \perp l$
Is complementary to	angles	$\angle 1$ is comp to $\angle 2$
Is congruent to	line segments, angles	$\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{BC}$
Is a brother	people	Mike is brother of Tom

There are three special properties that may exist for a given relation **R**.

Properties

- Reflexive property: a**R**a

Example: 5 = 5, equality of numbers has a reflexive property.

- Symmetric property: If a**R**b, then b**R**a.

Example: If $n \perp m$, then $m \perp n$, perpendicular lines have the symmetric property.

- Transitive property: If a**R**b and b**R**c, then a**R**c,

Example: If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$, congruence of angle is transitive.

Example: Given the line segment A- B- C- D give

a) An example of reflexive property

b) An example of the transitive property

c) An example of the symmetric property

Example: Does the relation "is a brother of " have

a) A reflexive property (consider one male)

b) A symmetric property (consider two males)

c) A transitive property (consider three males)

Example: Does the relation "is complementary to" have

a) A reflexive property (consider one angle)

b) A symmetric property (consider two angles)

c) A transitive property (consider three angles)